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CORRECTIONS TO

"Einstein metrics in dimension four"*

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1. In Lemma 2.5 (p. 432), a crucial hypothesis is missing: namely, the vector field w should also be assumed transverse to N at z, that is, $w(z) \notin T_zN$.

2. In formula (3.33) on p. 443, the matrix $\mathfrak{C}^*$ is *not* the transpose of $\mathfrak{C}$, but rather a "modified transpose" with the entries $c^*_{jk} = \varepsilon_j \varepsilon_k c_{kj}$.

3. Formula (4.27) on p. 447 has the wrong sign and should read

$$\nabla_w \nabla_v F \,-\, \nabla_v \nabla_w F \,+\, \nabla_{[v,w]} F \,=\, [R(v,w), F]\,. \tag{4.27}$$

Used on pp. 470 (bottom), 471 (top), 485 (near top), 514 (near bottom), 547 (middle of the page).

4. In Lemma 6.16 (p. 466), the assumption is not strong enough to yield the stated assertion. To correct this, the phrase:

and $\mathrm{s} = 0$ *at every point of* M*, so that* (M, g) *is anti-self-dual and its scalar curvature is identically zero.*

should be replaced by:

and $\mathrm{Ric} = 0$ *at every point of* M*, so that* (M, g) *is anti-self-dual and Ricci-flat.*

5. Formula (6.27) on p. 470 has the wrong sign and should read

$$d\xi_j + \xi_k \wedge \xi_l \,=\, -\,(\lambda_j +\, \mathrm{s}/12)\,\alpha_j \,-\, (\mathrm{E}\alpha_j \,+\, \alpha_j\mathrm{E})/2 \qquad \text{if} \quad \varepsilon_{jkl} = 1\,. \tag{6.27}$$

Also, in the proof of Lemmas 6.16 and 6.17 (bottom of p. 470), signs need to be corrected. Used on pp. 470 (bottom, twice), 477 (the line following formula (7.1)), 479 (second line after formula (7.16)), 569 (twice: line 6^+, and sixth line before Remark 20.2), 573 (line 8^+).

6. Formula (8.8) on p. 480 (last line) and the first line on p. 481 should read

$$g((\nabla_{e_b} R)(u, e_a)e_c, e_d) \,=\, \varepsilon\,[F_{ac}G_{bd} \,-\, F_{ad}G_{bc} \,-\, F_{bd}G_{ac} \,+\, F_{bc}G_{ad}] \tag{8.8}$$

for $a, b, c, d = 2, \ldots, n$, with $G_{ab} = F^c_a F_{cb}$.

7. On p. 481, assertion (ii) of Lemma 8.4 and the last sentence in its proof (lines 11 and 9 – 8 from below) are incorrect. The lines in question should be replaced by:

(ii) If the operator $F^2 : \mathcal{X} \to \mathcal{X}$ is not a multiple of F, then g is not locally symmetric.

and, respectively:

from (8.7). As for (ii), it follows from (8.8): if R were parallel, summing (8.8) against $v^a v^b$ for any $v \in \mathcal{X}$ we would obtain $Fv \wedge F^2 v = 0$ (cf. (2.15)). Every nonzero vector Fv in the image $F(\mathcal{X})$ thus would be an eigenvector of F, i.e., F restricted to $F(\mathcal{X})$ would equal a scalar times Id, which contradicts the assumption of (ii).

8. On p. 481, bottom line, the phrase $F^3 \neq 0$ should read: such that F^2 is not a multiple of F

9. At the very end of Remark 13.10 (p. 510, line 18 from above), equality $\mathbf{r}_j = \mathbf{e}_j$ should read

$$\mathbf{r}_j = \mathbf{e}_j + (\partial_j f)\,\mathbf{u}$$

10. The formula defining ξ on p. 512 (line 6 from above) is incorrect.

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*Chapter 4 of *Handbook of Differential Geometry, Vol. I* (edited by F.J.E. Dillen and L.C.A. Verstraelen), Elsevier Science B.V., 2000, pp. 419–707. I wish to thank Sung Young Lee and Gideon Maschler, who brought the errors listed here to my attention.

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11. In the final clause of Lemma 15.12 (p. 524), the stated assumptions are not strong enough to yield the assertion. To correct this, the phrase:

For $\mathcal{T}$ and $\mathcal{Z}$ related in this manner, $\mathcal{T}$ is nondegenerate as a subspace of the pseudo-Euclidean space V if and $\mathcal{Z}$ and the space (15.8) *together span $V/\mathcal{W}$.*

should be replaced with:

If $k = 1$ then, for $\mathcal{T}$ and $\mathcal{Z}$ related in this manner, $\mathcal{T}$ is nondegenerate as a subspace of the pseudo-Euclidean space V if $\mathcal{Z}$ and the space (15.8) *together span $V/\mathcal{W}$.*

12. In formula (16.24) on p. 531, the factor $(n-3)$ should be dropped; the correct formula is

$$\tilde{Z} = Z + W(df, \cdot, \cdot, \cdot) \tag{16.24}$$

13. In Remark 16.10 (p. 535, line 11 from above),

instead of: in view of Proposition 22.3(iii) in §22

read: in view of Proposition 22.4 in §22

14. At the end of Example 17.9 (p. 539, immediately before Remark 17.10), the following sentence should be deleted:

Note that, in this case, $[\ ]^{\mathrm{tang}}$ becomes redundant in formula (13.8), i.e., by (17.7), $\nabla_v w = \mathrm{D}_v A$, since $Ax \in x^\perp = T_x M$.

15. In formula (18.7) (p. 550), $\tilde{\mathrm{s}}$ should be replaced by $\tilde{\mathrm{s}}/4$.

16. In the proof of Lemma 18.9 (p. 552, line 13 from below),

instead of: By Corollary 11.2 we have read: By Corollary 11.3 we have

17. In the displayed formula preceding formula (18.18) (p. 553),

instead of: $= -\,2\kappa\varPhi$ read: $= -\,2\delta\kappa\varPhi$

18. At the end of the proof of Lemma 18.9 (p. 554, line 13 from above),

instead of: $\varepsilon\sigma h = \alpha\nabla w = \alpha^2(\nabla d\kappa) = \varepsilon\,\nabla d\kappa$ read: $\varepsilon\sigma h = -\,\alpha\nabla w = -\,\alpha^2(\nabla d\kappa) = \varepsilon\,\nabla d\kappa$

19. At the end of Remark 19.9 (p. 564, line 9 from above), formula $\lambda_a = -\,\varepsilon\mu_a/2$ (in the text) should read $\lambda_a = -\,\mu_a/2$.

20. In the third line of the proof of Lemma 28.6 (p. 606), the phrase 'where exp is' should be replaced by 'with $c = 0$, where *exp* is'.

21. In both displayed formulas immediately following formula (28.13) (p. 607), expression $(b-a)^{-2}|\dot\gamma|^2]$ should read $(b-a)^2|\dot\gamma|^2]$.

22. In Example 36.9 (p. 636), condition $1 \le k \le 8$ should be replaced by $k \in \{1,2\}$.