Extreme parts of the Khovanov complex

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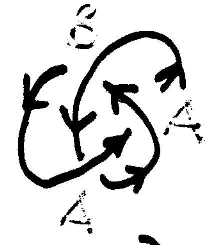
Y. Bae, H. Morton
"The spread and extreme terms of Jones polynomials"
J.K.T.R. <u>12</u> (2003) 359-373
math.GT/0012089

<u>Kauffman's state</u> s

$\alpha(s) = \#(\text{A-splittings})$

$\beta(s) = \#(\text{B-splittings})$

$\delta(s) = \#(\text{circles})$

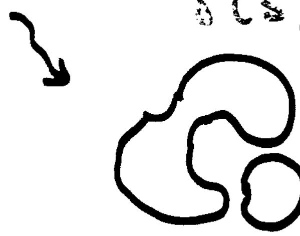


$n_+ = 0, \; n_- = 3$

$\alpha(s) = 2$

$\beta(s) = 1$

$\delta(s) = 2$



<u>Enhanced state</u> S. the state circles are colored by V_- or V_+

$\dim(S) := \beta(s) - n_-$

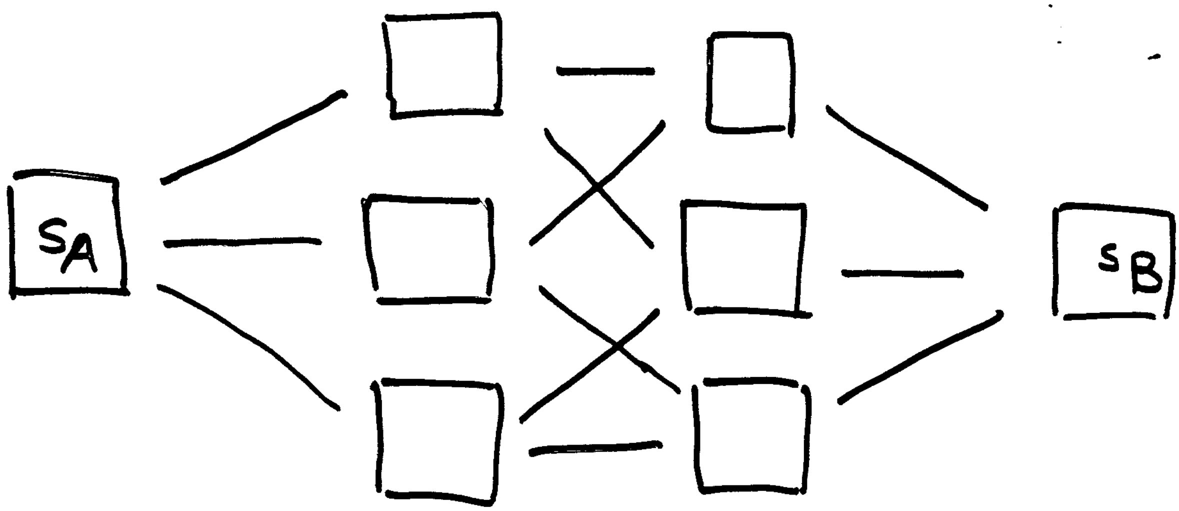
$\deg(S) := \#(V_+\text{-circles}) - \#(V_-\text{-circles}) + \beta(s) + n_+ - 2n_-$

$$-\delta(s) + \beta(s) + n_+ - 2n_- \le \deg(S) \le \delta(s) + \beta(s) + n_+ - 2n_-$$

<u>Extreme states</u>

S_A all A-splittings, all V_-

S_B all B-splittings, all V_+



If $\mathcal{S}'$ is obtained from $\mathcal{S}$ by changing of an A-splitting of $\mathcal{S}$ to B-splitting then

$$-\delta(s)+\beta(s)+n_+-2n_- \leq -\delta(s')+\beta(s')+n_+-2n_-$$

$$\delta(s)+\beta(s)+n_+-2n_- \leq \delta(s')+\beta(s')+n_+-2n_-$$

$$\Downarrow$$

$$\boxed{-\delta(s_A)+n_+-2n_- \leq \deg(\mathcal{S}) \leq \delta(s_B)+2n_+-n_-}$$

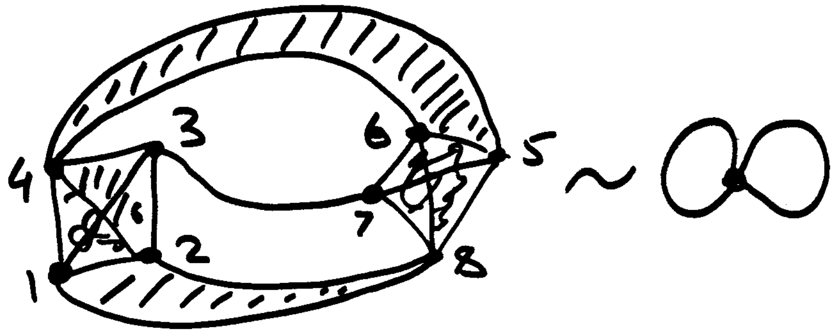
Independence complex of a graph Γ.

0 - simplices = vertices of Γ

k - simplices = subsets of k independent vertices of Γ

$\mathrm{Ind}(\Gamma)$

For example:



Theorem.

1. Extreme top Khovanov complex coincide with the augmented cochain complex of $\mathrm{Ind}(\Gamma_B)$
2. Exteme Bottom Khovanov complex coinside with the augmented chain complex of $\mathrm{Ind}(\Gamma_A)$

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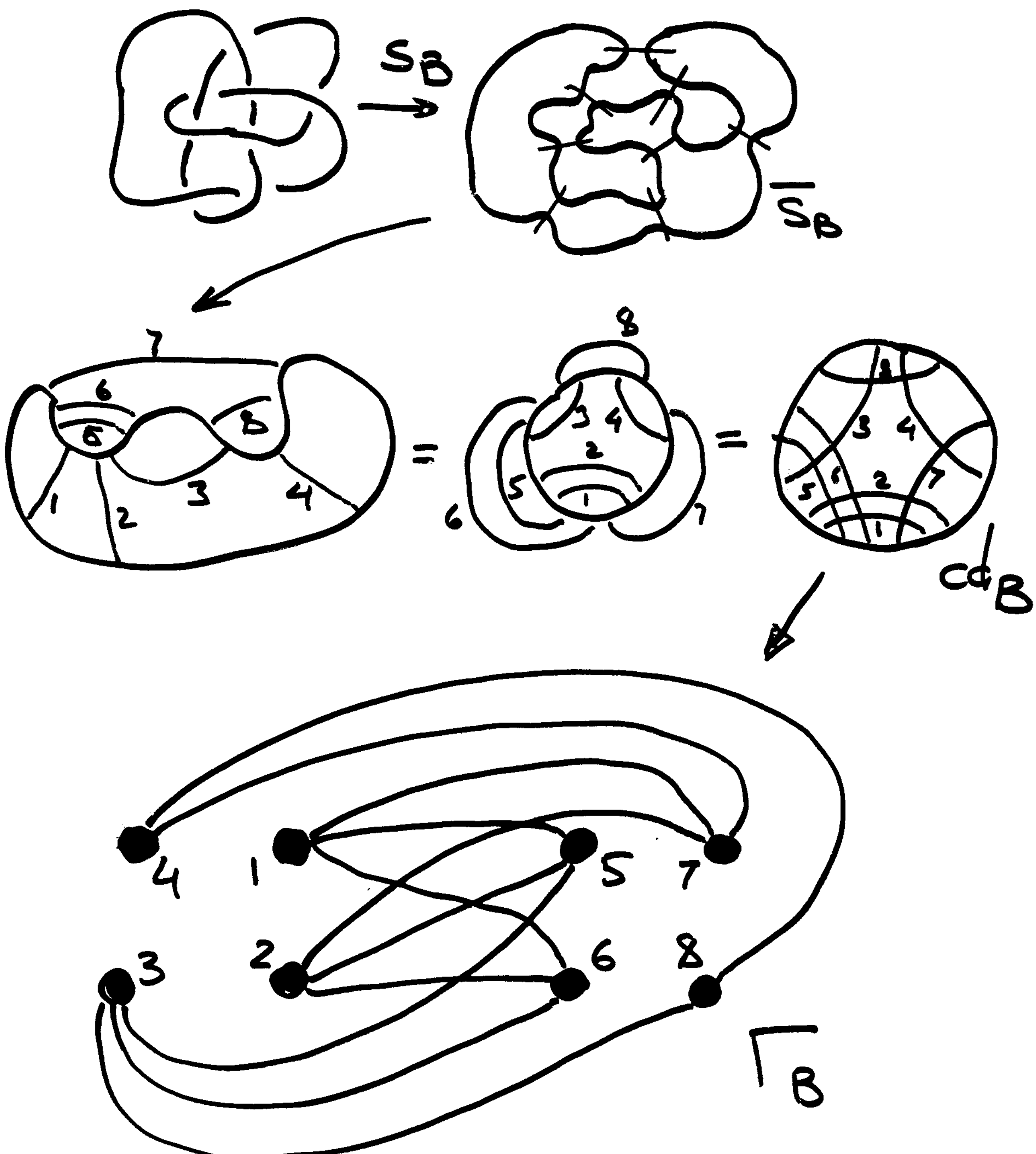

Bae-Morton: $a_{s_B} = (-1)^{\delta(s_B)-1} \sum_C (-1)^{|C|}$

C is a subset of <u>independent</u> <u>vertices</u>

of Γ_B

Conjecture.

If Γ is the intersection graph of a chord diagram then $\mathrm{Ind}(\Gamma)$ is homotopy equivalent to a wedge of spheres of the same dimension