Math Circle, Winter 2008<br>January 20, 2008<br>www.math.ohio-state.edu/ross/MathCircle

1. Describe the result of cutting a band with $m$ twists along $n$ lines parallel to its edge.
2. Find the genus and the number of boundary components of the surface.

3. Consider the surface consisting of two discs and $m$ twisted bands between them. Is it orientable? Find the genus and the number of boundary components.
4. Find the numbers of cross-caps and the numbers of boundary components for the following surfaces.

5. Prove that the Euler characteristics $\chi$ and the genus $g$ of a closed orientable surface are related as $\chi=2-2 g$.
6. Prove that the Euler characteristics $\chi$ and the number of cross-caps $\mu$ of a closed nonorientable surface are related as $\chi=2-\mu$.
7. Find a minimal triangulation of a torus.
8. Find a minimal triangulation of a Klein bottle.
