Lecture 2: Jones polynomial and Thistlethwaite's theorem.

May 19, 2016

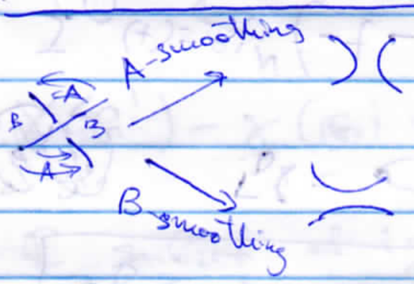
Knots and links  diagrams

Reidemeister theorem

(0) Planar isotopy (I)  (II)

(III) 

Kauffman bracket



A state S is a choice of either A- or B-smoothing at every crossing

$\alpha(S) :=$ # A-smoothings

$\beta(S) :=$ # B-smoothing

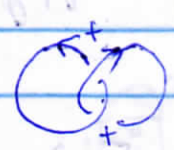
$\delta(S) :=$ # circles of S

Def. $$[L](A,B,d) := \sum_{S \text{ states}} A^{\alpha(S)} B^{\beta(S)} d^{\delta(S)-1}$$

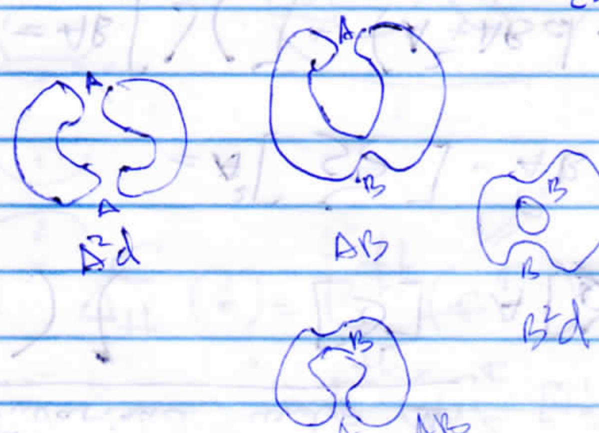
Writhe of oriented L 

$w(L) = \sum_x \varepsilon_x$

$$J_L(t) := (-1)^{w(L)} t^{3w(L)/4} [L]\left(t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2}\right)$$

Example $L =$  $w(L) = 2$

$[L] = A^2 d + 2AB + B^2 d$



$$J_L(t) = t^{3/2}\left(t^{-1/2}(-t^{1/2} - t^{-1/2}) + 2 + t^{1/2}(-t^{1/2} - t^{-1/2})\right)$$

$$= t^{3/2}(\cancel{-1} - t^{-1} \cancel{+2} - t \cancel{-1})$$

$$= -t^{1/2} - t^{5/2}$$

Invariance under II

$$[\ ] = A[\ ] + B[\ ]$$

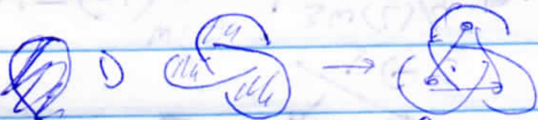
$$= A^2[\ ] + AB[\ ] + BA[\ ] + B^2[\ ]$$

$$= AB[\ ] + (A^2 + ABd + B^2)[\ ] = [\ ]$$

if $B = A^{-1}$

and $d = -A^2 - B^2$

Thistlethwaite's Theorem



D $\to$  $=: \Gamma_D \to T_D(x,y)$

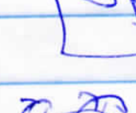
$$= \sum_{F \subseteq E(\Gamma)} (x-1)^{r(\Gamma)-r(F)} (y-1)^{n(F)}$$

$r(F) = |V(F)| - |F|$

$n(F) = |F| - r(F)$

Up to sign and a power of t (for alternating links)

$$\boxed{J_D(t) = \pm t^N \, T_{\Gamma_D}(-t, -t^{-1})}$$



$\Gamma_D =$  $\quad T_{\Gamma(D)} = T_{\bullet-\bullet} + T_{\circ} = xT_{\cdot} + yT_{\cdot}$

$= x + y$

$$J(t) = \pm t^N(-t - t^{-1}) = -t^{5/2} - t^{1/2}$$

$+N = 3/2$