On the extended bracket polynomial for virtual knots and links.

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Sunday, April 30, 2017
11:35 — 12:00

Ben O’Connor

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REU-like Working Group **Knots and Graphs** [Summer 2012]

https://people.math.osu.edu/chmutov.1/wor-gr-su12/wor-gr.htm

Louis Kauffman. *An extended bracket polynomial for virtual knots and links. J. Knot Theory Ramifications*, **18**(10) (2009) 1369–1422.

Lou Kauffman. Algorithm.

Louis Kauffman. *An extended bracket polynomial for virtual knots and links. J. Knot Theory Ramifications*, **18**(10) (2009) 1369–1422.

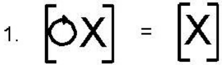
$$\langle\langle K\rangle\rangle := \sum_{S} A^{\alpha(S)}\, B^{\beta(S)}\, \delta^{||S||-1}[S]\ ,$$

where $[S]$ is a sum of reduced states (flat virtual graphs).

Algorithm of simplification.

(1) 

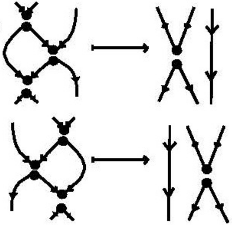
(2) Perform all single oriented loop simplifications of type 1.



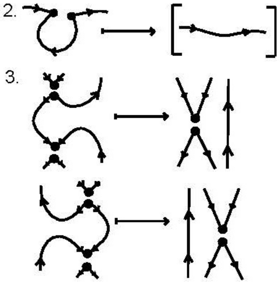
Lou Kauffman. Algorithm.

(3) Make all replacements of type C. Repeat the previous step if necessary.

C.



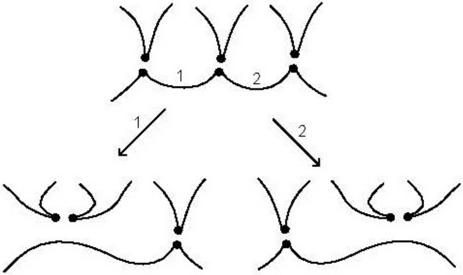
(4)



Make all replacements of types 2 and 3. Repeat the previous steps if necessary. If there are a multiplicity of replacements of type 3 available, make each replacement and take the sum of all of them divided by the number of such replacements.

Lou Kauffman. Algorithm.

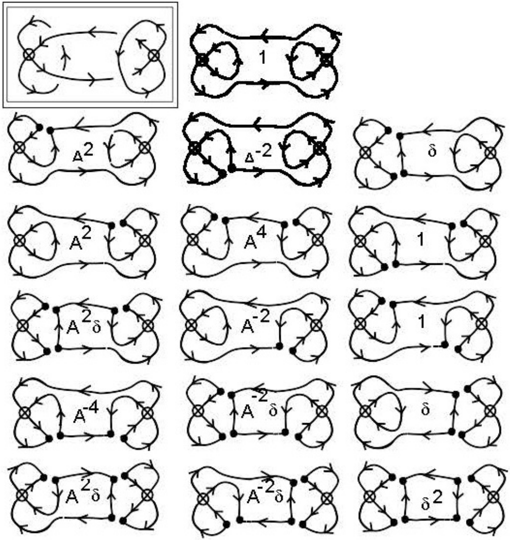
Multiplicity:



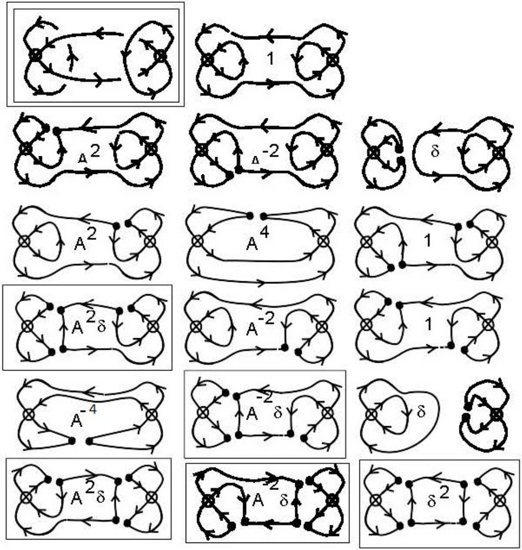
(5) 

When there are no more simplifications available, replace each remaining disoriented smoothing with a graphical vertex.

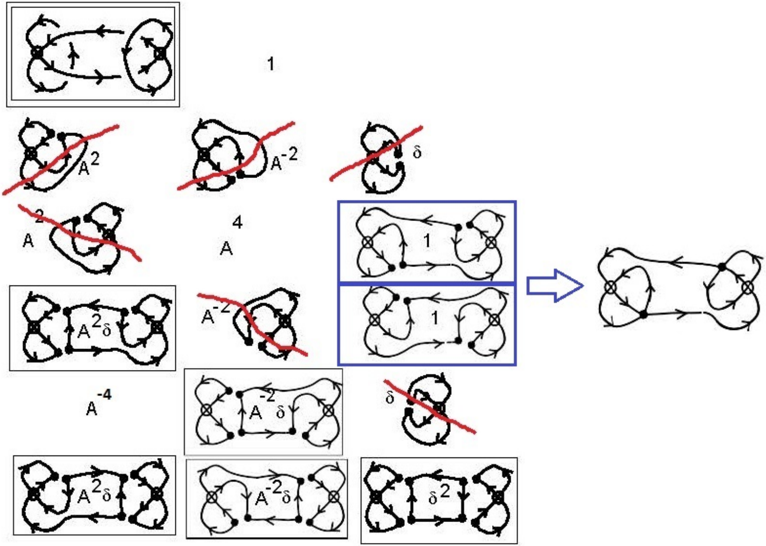
Kishino diagram.



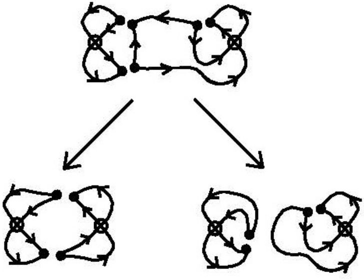
Kishino diagram.



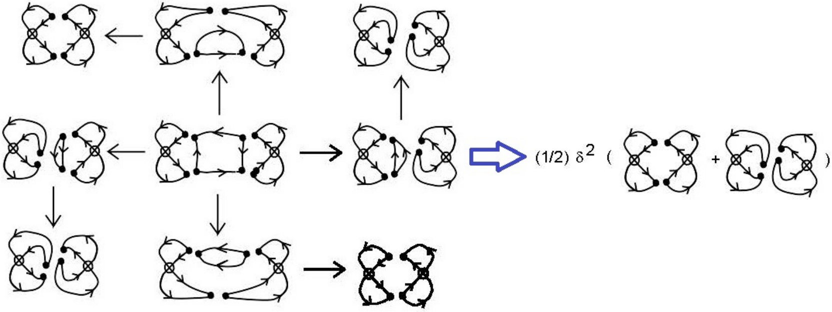
Kishino diagram.



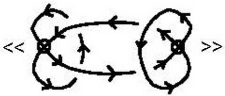
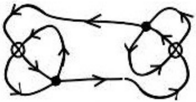
Kishino diagram. Reducing in multiplicity.

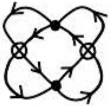


Kishino diagram. Reducing in multiplicity.

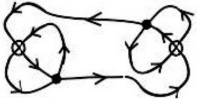
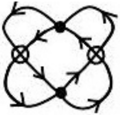


Kishino diagram. Answer.

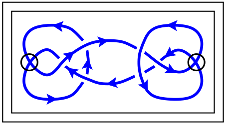
 $= 1 + A^4 + A^{-4} + 2$ 

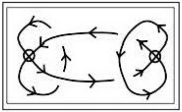
$-(1/2)\delta^2 \Big($  +   $\Big)$

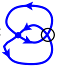
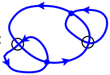
$$\boxed{= 1 + A^4 + A^{-4} + 2G_4 - (1/2)\delta^2(G_2 + G_3^2)}$$

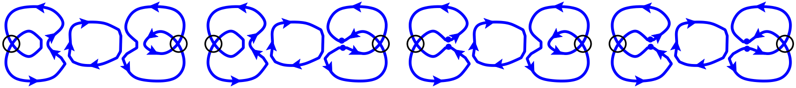
$G_4 :=$  $G_2 :=$  $G_3 :=$ 

Ben O'Connor. Counterexample.

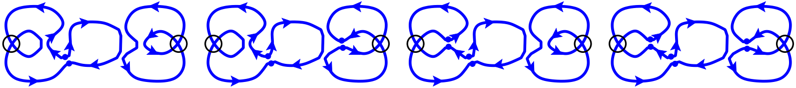




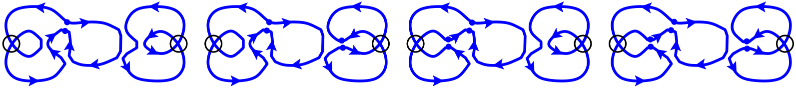
$G_3 :=$  $G_4 :=$ 



$$\delta^2 + A^2\delta^2 G_3 + A^{-2}\delta^2 G_3 + \delta^2 G_3^2 = \delta^2 - \delta^3 G_3 + \delta^2 G_3^2$$



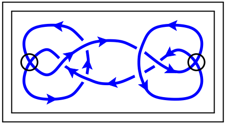
$$A^2\delta + A^4\delta G_3 + \delta G_3 + A^2\delta G_3^2$$

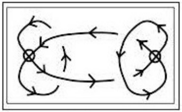


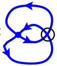
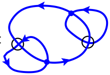
$$A^{-2}\delta + \delta G_3 + A^{-4}\delta G_3 + A^{-2}\delta G_3^2$$

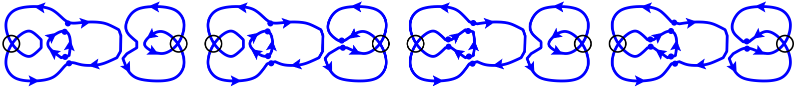
Slide net result is ZERO!

Ben O'Connor. Counterexample.

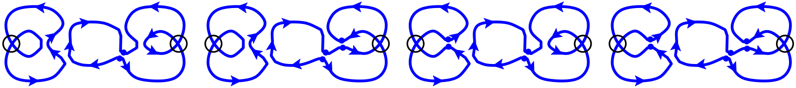




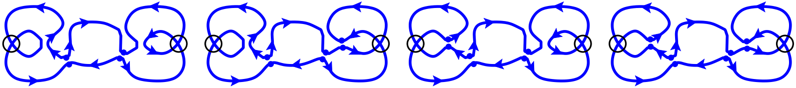
$G_3 :=$  $G_4 :=$ 



$$\delta^2 + A^2\delta^2 G_3 + A^{-2}\delta + \delta G_3$$



$$A^2\delta + A^4\delta G_3 + \delta G_3 + A^2\delta G_3^2$$

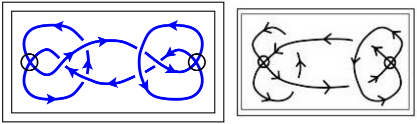


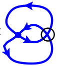
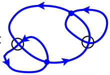
$$A^4 + A^6 G_3 + A^2 G_3 + A^4$$

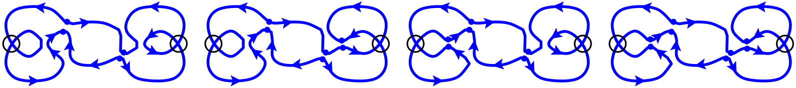
Slide net result is

$$2A^4 + A^6 G_3 + A^4\delta G_3 + A^2\delta^2 G_3 + A^2 G_3 + 2\delta G_3 + A^2\delta G_3^2$$

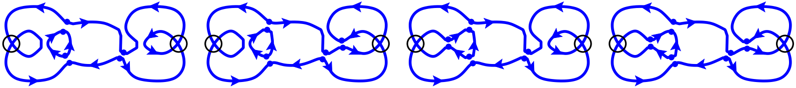
Ben O'Connor. Counterexample.



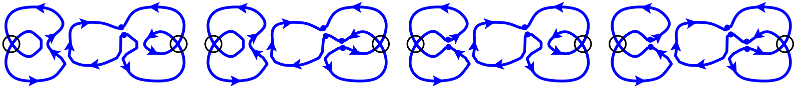
$G_3 :=$  $G_4 :=$ 



$$1 + A^2 G_3 + A^{-2} G_3 + G_4$$



$$A^2\delta + A^4\delta G_3 + 1 + A^2 G_3$$

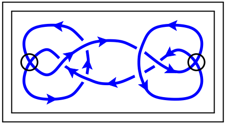
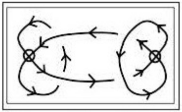


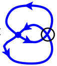
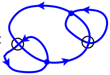
$$A^{-2}\delta + \delta G_3 + A^{-4}\delta G_3 + A^{-2}\delta G_3^2$$

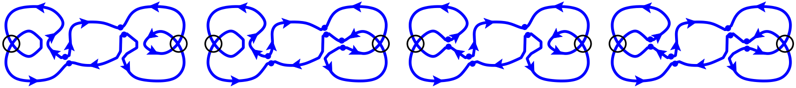
Slide net result is

$$2 - \delta^2 + A^2 G_3 + A^4\delta G_3 + A^{-4}\delta G_3 + A^{-2}\delta G_3^2 + G_4$$

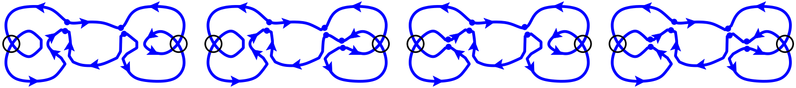
Ben O'Connor. Counterexample.

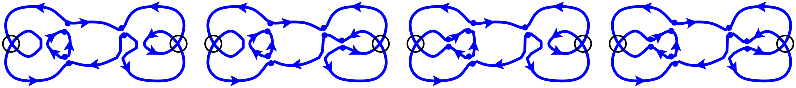
$G_3 :=$  $G_4 :=$ 



$$1 + A^2 G_3 + A^{-2} G_3 + G_4$$



$$A^{-4} + A^{-2} G_3 + A^{-6} G_3 + A^{-4}$$

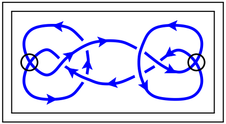


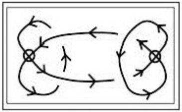
$$A^{-2}\delta + \delta G_3 + A^{-4} + A^{-2} G_3$$

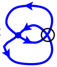
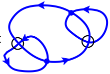
Slide net result is

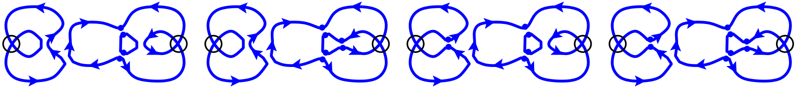
$$1 + 3A^{-4} + A^{-2}\delta + \delta G_3 + 3A^{-2} G_3 + A^{-6} G_3 + A^2 G_3 + G_4$$

Ben O'Connor. Counterexample.

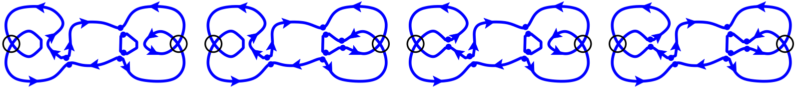




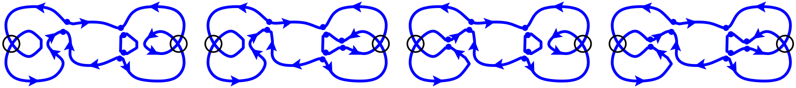
$G_3 :=$  $G_4 :=$ 



$$\delta^2 + A^2\delta + A^{-2}\delta^2 G_3 + \delta G_3$$



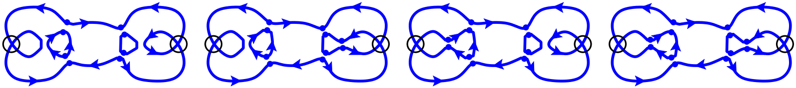
$$A^2\delta + A^4 + \delta G_3 + A^2 G_3$$



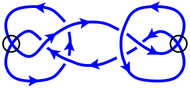
$$A^{-2}\delta + 1 + A^{-4}\delta G_3 + A^{-2} G_3$$

Slide net result is $1 + A^4 + A^2\delta + \delta G_3 + A^{-2}\delta^2 G_3 + A^{-4}\delta G_3$

Ben O'Connor. Counterexample.

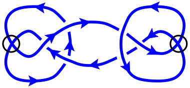


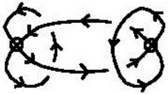
$$\delta^2 + A^2\delta + A^{-2}\delta + 1 = 1$$

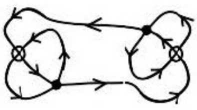
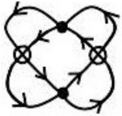
$\left\langle\!\left\langle$  $\right\rangle\!\right\rangle = 0$

$$\begin{aligned}
&+2A^4 + A^6 G_3 + A^4\delta G_3 + A^2\delta^2 G_3 + A^2 G_3 + 2\delta G_3 + A^2\delta G_3^2\\
&+2 - \delta^2 + A^2 G_3 + A^4\delta G_3 + A^{-4}\delta G_3 + A^{-2}\delta G_3^2 + G_4\\
&+1 + 3A^{-4} + A^{-2}\delta + \delta G_3 + 3A^{-2} G_3 + A^{-6} G_3 + A^2 G_3 + G_4\\
&+1 + A^4 + A^2\delta + \delta G_3 + A^{-2}\delta^2 G_3 + A^{-4}\delta G_3\\
&+1 = 1 + A^4 + A^{-4} + 2G_4 - \delta^2 G_3^2
\end{aligned}$$

Counterexample.

$\langle\langle$  $\rangle\rangle = 1 + A^4 + A^{-4} + 2G_4 - \delta^2 G_3^2$

$<<$  $>> \; = 1 + A^4 + A^{-4} + 2G_4 - (1/2)\delta^2 (G_2 + G_3^2)$

$G_4 :=$  $G_2 :=$  $G_3 :=$