

## SOME INTEGRALS

**A few more nonelementary integrals.** The following is a list of some nonelementary integrals:

$$(1) \quad \int \frac{e^x}{x} dx = \text{Ei}(x); \quad \int e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \text{erf}(x);$$

$$F(\sin \phi; k) = \int_0^\phi \frac{1}{\sqrt{1 - k^2 \sin^2 s}} ds, \quad E(\sin \phi; k) = \int_0^\phi \sqrt{1 - k^2 \sin^2 s} ds$$

Ei is called the “exponential integral”, erf is the “error function”,  $F$  is an “incomplete elliptic integral of the first kind”,  $E$  is an “incomplete elliptic integral of the second kind”. The exercise below is meant to help you practice integration by parts and substitution.

**Exercise 1** (28p). *Express the following integrals in terms of elementary functions and the functions above.*

$$(1) \quad \int x^2 e^{-x^2} dx$$

$$(2) \quad \int x^{-1} e^{-x^2} dx$$

$$(3) \quad \int \ln(\ln x) dx$$

$$(4) \quad \int \sqrt{\frac{4-x^2}{1-x^2}} dx$$

$$(5) \quad \int \frac{1}{\sqrt{x^4 - 5x^2 + 4}} dx$$

$$(6) \quad \int \sqrt{\frac{4-x}{x(1-x)}} dx$$

$$(7) \quad \int \frac{dx}{\sqrt{4x^3 + 7x^2 + 3x}}$$