PROBLEM SET 1

(1) Let K be an arbitrary compact set in $\mathbb{C}$. Show that there is a compact Hausdorff space X and a function $f \in C(X)$ such that $\sigma(f) = K$.

(2) Let M be an n by n matrix and let $\mathcal{A}$ be the Banach algebra generated by M in the operator norm $\|M\| = \sup_{x\in\mathbb{R}^n, \|x\|=1} |M(x)|$, where $|\cdot|$ is the Euclidian distance. Describe the set of maximal ideals and of multiplicative functionals on $\mathcal{A}$.

(3) Let B be the Banach space $C[0,1]$ with the usual sup norm, and $\mathcal{P}$ be the operator $f \mapsto \int_0^x f$. Check that $\mathcal{P}$ is bounded. Let $\mathcal{A}$ be the Banach algebra generated by $\mathcal{P}$. Find $\sigma(\mathcal{P})$ in $\mathcal{A}$.