2012 AMS JOINT MATHEMATICS MEETINGS
SPECIAL SESSION ON HOMOTOPY THEORY

Friday, January 6: 8-10:50AM, Boston Marriott 4th floor, Provincetown.

8-8:20AM: William Kronholm

On the equivariant cohomology of Stiefel manifolds

Let G be a group of order 2. A Rep(G)-complex structure is put on the special orthogonal group with a particular action of G. This cell structure is then used to put a cell structure on the Stiefel manifold of k-frames in a real representation of G. These structures are then used to investigate the $RO(G)$-graded cohomology of the special orthogonal groups, Stiefel manifolds, and Grassmann manifolds.

8:30-8:50AM: Daniel Dugger

Cohomology of equivariant Grassmannians and motivic characteristic classes for quadratic bundles

The talk will present a computation of the $RO(G)$-graded Eilenberg-MacLane cohomology of $Z/2$-equivariant real Grassmannians. I will explain how this connects to a theory of characteristic classes for quadratic bundles with values in motivic cohomology.

9-9:20AM: Anna Marie Bohmann

Global equivariant homotopy theory

Much recent work has shown that equivariant homotopy theory can give insight into the non-equivariant world. While concrete calculations focus on specific groups, many familiar objects in homotopy theory have (or we would like them to have) equivariant generalizations that feel "natural." One way of stating such naturality is by asking how these generalizations fit together across different groups of equivariance. Global equivariant homotopy theory is the study of spectra that vary naturally in the group of equivariance. Change of groups is an important tool in existing calculations, and one might hope that some calculations work globally and not just one group at a time; it is also interesting to determine the precise functoriality of calculations such as the Segal conjecture or the Atiyah-Segal completion theorem that are already known to work globally. I will discuss the basic ideas of what we mean by "global" spectra and explain how these notions capture the naturalness we see in familiar spectra such as complex cobordism and K-theory, but don't see for Eilenberg-MacLane spectra.

9:30-9:50AM: Betrand Guillou

G-spectra and equivariant commutativity

I will discuss a model for the equivariant stable homotopy category in which the objects are spectral functors on a suitable domain category. One of the central ingredients is equivariant infinite loop space theory, whose input is appropriately commutative equivariant data. I will discuss this structure and consequences thereof from several viewpoints.

10-10:20AM: Kári Ragnarsson

The Segal conjecture in homotopical group theory

For a finite p-group P and a finite group G, the Segal conjecture implies a description, due to Lewis-May-McClure, of the spectrum of stable maps from BP to the p-completion of BG as a wedge sum of suspension spectra. In unpublished work, Lannes showed that when P has order p, one can replace BG with a space X that shares some homotopy characteristics with the classifying space of a finite group, and obtain a similar description. We will discuss this work and show how, using iterated homotopy fixed points, one can obtain a description of the spectrum of stable maps from BP to X for a general finite p-group P. The allowable spaces X in this setting include p-compact groups and p-local finite groups, and thus we obtain a version of the Segal conjecture for those spaces.

10:30-10:50AM: Matthew Gelvin

A homotopical version of p-local finite groups

A p-local finite group is an algebraic model for the p-completed classifying space of a finite group. Miller conjectured that such spaces could be understood as a homotopical phenomenon, in terms of maps between a given space and the classifying space of a finite p-group. In this talk we will describe these concepts in more detail and give an outline of a proof of Miller's conjecture.

Saturday, January 7 AM: 8-10:50AM, Boston Marriott 4th floor, Provincetown.

8-8:20AM: Ricardo Andrade

Discrete models for configuration spaces

The objective of this talk is to give simple pictorial descriptions of categories whose classifying spaces are equivalent to configuration spaces of familiar manifolds.

8:30-8:50AM: Samuel Isaacson

Dendroidal sets and symmetric monoidal infinity categories

Moerdijk, Berger, and Cisinski have developed several homotopy theories of ∞-operads based up on the combinatorics of "dendroidal sets." Their work generalizes many of the homotopy theories of $(\infty, 1)$-categories and is combinatorially attractive. In my talk I'll discuss some progress on analogues of some 1-categorical notions in the dendroidal world.

9-9:20AM: Romie Banerjee

Categories of Modules and their Deformations

Using Quillen-Lurie deformation theory formalism we develop an obstruction theory for studying the stable ∞ category of modules over a given geometric ∞ stack, and produce a more general version of the Thomason-Trobaugh localization theorem for triangulated categories. This helps us identify a large class of perfect geometric stacks. Applications include Grothendieck duality.

9:30-9:50AM: David Ayala

Weak n-categories are sheaves on $d \leq n$-manifolds

This talk will present a geometric setting equivalent to the theory of weak n-categories in the sense of Rezk. Specifically, I will explain how a weak n-category is indexed by the space of configurations of points in the diagram of projections

$$\mathbb{R}^n \to \mathbb{R}^{n-1} \to \cdots \to \mathbb{R}^0;$$

and as so, from a weak n-category we will construct a sheaf on a site of iterated submersions of framed n-manifolds which are equipped with a configuration of points. Applied to E_n-algebras, this construction is chiral homology. A theorem will be stated that this construction implements an equivalence between weak n-categories and sheaves on this site. This work is joint with Nick Rozenblyum.

10-10:20AM: Nick Rozenblyum

Manifolds, Higher Categories and Topological Field Theories

I will describe an ongoing project with David Ayala to describe weak n-categories with adjoints as sheaves on the site of n-manifolds together with the additional data of transversality. Such a sheaf gives an n-dimensional topological field theory which generalizes topological chiral homology for (twisted) E_n algebras and should be related to the blob complex.

10:30-10:50AM: Christopher Schommer-Pries

On the uniqueness of the homotopy theory of higher categories

We propose axioms that a quasicategory should satisfy to be considered a reasonable homotopy theory of (∞, n)-categories. This axiomatization requires that a homotopy theory of (∞, n)-categories, when equipped with a small amount of extra structure, satisfies a simple, yet surprising, universal property. We further prove that the space of such quasicategories is homotopy equivalent to $B(\mathbb{Z}/2)^{\times n}$. This generalizes a theorem of Töen when $n = 1$, and it verifies two conjectures of Simpson. In particular, any two such quasicategories are equivalent. We also provide a large class of examples of models satisfying our axioms, including those of Joyal, Kan, Lurie, Simpson, and Rezk. This is joint work with Clark Barwick.

Saturday, January 7 PM: 1-5:50PM, Boston Marriott 4th floor, Provincetown.

1-1:20PM: André Joyal*, Matthieu Anel

A general bar-cobar duality

We extend Sweedler's theory of algebras and coalgebras to operads and cooperads. We show for that the category of cooperads is symmetric monoidal closed and that the category of operads is enriched over it. This is true generally for operads and cooperads enriched in any symmetric monoidal locally presentable category, and in particular for differential graded operads and cooperads. We then formulate the bar-cobar duality for operads and cooperads in this setting.

1:30-1:50PM: John Harper

Localization and completion with respect to topological Quillen homology

Quillen's derived functor notion of homology provides interesting and useful invariants in a wide variety of homotopical contexts. For instance, in Haynes Miller's proof of the Sullivan conjecture on maps from classifying spaces, Quillen homology of commutative algebras (André-Quillen homology) is a critical ingredient. Working in the topological context of symmetric spectra, this talk will introduce several recent results on localization and completion with respect to topological Quillen homology of commutative ring spectra (topological André-Quillen homology), E_n ring spectra, and operad algebras in spectra. This includes homotopical analysis of a completion construction and strong convergence of its associated homotopy spectral sequence—analogous to results by Bousfield-Kan on the R-completion of spaces—and a description of a point-set model of the derived comonad (or cotriple) that coacts on the object underlying topological Quillen homology; in other words, topological Quillen homology is a coalgebra over this comonad. Several of the results are joint work with Michael Ching and Kathryn Hess.

2-2:20PM: Michael Shulman

Cell complexes and inductive definitions

In recent work of Voevodsky, Awodey, and others, it has emerged that Martin-Löf's constructive type theory, originally conceived as a computational foundation for mathematics, can naturally be interpreted in homotopy theory. In particular, many standard theorems in homotopy theory can be proven inside of type theory, and thereby fully verified for correctness quite easily by a computer. This promises a fruitful interplay between the two disciplines, and potentially a new foundation for mathematics which is at once "homotopical" and "computational". What is still missing, however, is a way to construct, in type theory, familiar spaces such as spheres, tori, manifolds, classifying spaces, Postnikov towers, and so on, which in homotopy theory we usually describe using cell complexes. In joint work with Peter Lumsdaine, we have shown that the type-theoretic notion of *inductive definition* admits a generalization that naturally includes all such constructions. I will describe the resulting notion, assuming no background in type theory, and explain how it matches homotopy-theoretic cell complexes and Quillen's small object argument.

2:30-2:50PM: Martin Frankland*, Hans-Joachim Baues
Non-realizable 2-stage Π*-algebras*
It is a classic fact that Eilenberg-MacLane spaces exist and are unique up to weak equivalence. However, one cannot always find a space with two non-zero homotopy groups and prescribed primary homotopy operations. Using work of Baues and Goerss, we will present examples of non-realizable 2-stage Π-algebras, focusing on the stable range.

3-3:20PM: Angelica Osorno
Stable homotopy 1-types and symmetric Picard groups
It is a classical result that groupoids model homotopy 1-types, in the sense that there is an equivalence between the homotopy categories, via the classifying space and fundamental groupoid functors. We extend this result to stable homotopy 1-types and symmetric Picard groupoids, that is, symmetric monoidal groupoids in which every object has a weak inverse. Using an algebraic description of symmetric Picard groupoids, we identify the Postnikov data associated to a stable 1-type; the abelian goups π_0 and π_1, and the unique k-invariant. We relate this data to the exact sequences of Picard groupoids developed by Vitale.

3:30-3:50PM: Niles Johnson
Obstruction theory for E_∞ maps
We take an obstruction-theoretic approach to the question of algebraic structure on spectra. At its heart, this is an application of the Bousfield-Kan spectral sequence adapted for general operadic structure in a range of topological categories. This talk will focus on examples from rational homotopy theory which illustrate the obstructions to rigidifying homotopy algebra maps between differential graded algebras to strict algebra maps. In the topological context, these provide explicit examples of H_∞ maps which cannot be rigidified to E_∞ maps.

4-4:20PM: Tyler Lawson
Truncated Brown-Peterson spectra
Truncated Brown-Peterson spectra $\mathrm{BP}\langle n\rangle$ and related Johnson-Wilson spectra $\mathrm{E}(n)$ play an important role in Ausoni and Rognes' program to study algebraic K-theory chromatically. This talk will focus on two problems: giving highly

structured multiplication to these spectra, and deciding what the definition of them should be.

4:30-4:50PM: John Lind

Higher Geometry and Algebraic K-theory

A cohomology theory E is particularly useful when we can understand its cocycles $E^*(X)$ in terms of geometric objects associated to the space X. A basic example is the description of topological K-theory in terms of complex vector bundles. I will give an analogous interpretation of cocycles for $E = K(R)$, the algebraic K-theory of an associative ring spectrum R, in terms of bundles of R-modules over X. The main technological development is the use of diagram spaces. Diagram spaces are a symmetric monoidal model for the category of spaces in which A_∞ spaces are strict monoids. This provides a theory of "principal G-bundles" when G is an A_∞ space. The delooping BG classifies principal G-bundles, and the description of $K(R)$-theory follows from the case of $G = GL_n(R)$.

5-5:20PM: Paul Frank Baum

K-homology and index theory: Beyond ellipticity

This talk will indicate how K-homology can be used to extend the Atiyah-Singer index formula to a naturally arising class of non-elliptic operators. K-homology is the dual theory to K-theory – i.e. K-homology is the homology theory determined by the Bott K-theory spectrum. For a finite CW complex X, the K-homology of X can be defined via functional analysis and this gives the Kasparov groups $KK^*(C(X);\mathbb{C})$. A definition in the spirit of bordism theory uses K-cycles (M, E, φ) where M is a compact Spin^c manifold without boundary, E is a $\mathbb{C}$ vector bundle on M, and φ is a continuous map from M to X.

$$\varphi : M \to X$$

Denote the K-cycle version of K-homology by $K_*^{top}(X)$. The BD(Baum-Douglas) isomorphism

$$\mu\colon K_*^{top}(X) \to KK^*(C(X);\mathbb{C})$$

provides a framework for extending Atiyah-Singer beyond elliptic operators. The talk will first give the basic definitions, and will then show how the BD framework applies to a naturally arising class of hypoelliptic (but not elliptic) operators on contact manifolds. The above is joint work with Erik van Erp.

5:30-5:50PM: Vesna Stojanoska

Duality and Topological Modular Forms

We explain how a homotopy-theoretic self-duality of the spectrum of topological modular forms originates from a Grothendieck-Serre duality for moduli stacks of elliptic curves.