SATELLITE CONFERENCE ON HOMOTOPY THEORY AT MIT THURSDAY, JANUARY 5, 2012

1-1:20PM: Naeem Ahmad

Bordism of complex N-Spin manifolds with semifree circle actions

In this talk, we will discuss the complex N-Spin bordism groups of semifree circle actions and elliptic genera of level N.

The notion of complex N-Spin manifolds (or simply N-manifolds) was introduced by Höhn in [G. Höhn, *Komplex elliptische Geschlechter und S^1-äquivariante Kobordismustheorie,* Diplomarbeit, Bonn, 1991.]. Let the bordism ring of such manifolds be denoted by $\Omega_*^{U,N}$ and the ideal in $\Omega_*^{U,N} \otimes \mathbb{Q}$ generated by bordism classes of connected complex N-Spin manifolds admitting an effective circle action of type t be denoted by $I_*^{N,t}$. Also, let the elliptic genus of level n be denoted by φ_n. It is conjectured in [ibid.] that

$$I_*^{N,t} = \bigcap_{\substack{n|N \\ n\nmid t}} \ker(\varphi_n).$$

We will give a complete bordism analysis of rational bordism groups of semifree circle actions on complex N-Spin manifolds via traditional geometric techniques. We will use this analysis to give a determination of the ideal $I_*^{N,t}$ for several N and t, and thereby verify the above conjectural equation for those values of N and t. More precisely, we will verify that the conjecture holds true for all values of t with $N \le 9$, except for case $(N,t) = (6,3)$ which remains undecided. Moreover, machinery developed in our work furnishes a mechanism with which to explore the ideal $I_*^{N,t}$ for any given values of N and t.

1:30-1:50PM: Man Chuen Cheng

A Duality Theorem for Differentiable Stacks with respect to Morava K-theory

It was shown by Greenlees and Sadofsky that the classifying space of a finite group is self-dual with respect to Morava K-theory. This duality is induced by a transfer map. By regarding these classifying spaces as the homotopy types of certain differentiable stacks, I will explain how their construction can be viewed as a stack version of Spanier-Whitehead type construction and extend their results to a $K(n)$-version of Poincare duality for more general stacks.

2-2:20PM: Nathaniel Rounds

What is the algebraic structure of topological manifolds?

Can we associate an algebraic structure to a manifold such that this structure up to equivalence determines the manifold up to homeomorphism? If we replace the word "homeomorphism" with the word "homotopy equivalence", the answer is yes. We will describe various algebraic structures on a manifold's chains and cochains, all of which are known to be homotopy invariant. We will suggest, however, that the missing idea is that of algebraic locality. The various algebraic structures that we associate to a manifold are all local in an appropriate sense, but the inverse to the Poincaré duality map need not be local. We will show, using Ranicki's algebraic surgery, that considering the inverse to the Poincaré duality map leads to a topological invariant of manifolds. We will end with a (still partially conjectural) synthesis of all these ideas which gives an affirmative answer to the opening question.

2:30-2:50PM: Greg Chadwick

Structured Orientations of Thom Spectra

Given a map of ring spectra out of the complex cobordism spectrum MU, we can ask whether it may be represented by an E_n map. For a complex oriented ring spectrum E, ring maps from MU to E have been described by Quillen. When the target E is an E_∞ ring spectrum and in particular MU, E_n maps live in the unit spectrum cohomology of a cover of the classifying space BU. For E_2 or E_4 ring maps this cohomology is readily computable and demonstrates every self ring map of MU is E_2. This shows the Brown-Peterson spectrum BP is E_2.

3-3:30PM: Coffee break

3:30-3:50PM: Dev Sinha

The two-local cohomology rings of symmetric groups

We present an additive basis of "skyline diagrams" and multiplication rules for the two-local cohomology rings of symmetric groups. We start with two simpler kinds of rings to which these map, namely the mod-two cohomology of symmetric groups and the symmetric invariants of the integral cohomology of RP^∞.

4-4:20PM: David T. Oury

The Anodyne Theorem in Model Category Theory

The goal of this talk is to define a class of theorems which we call Anodyne Theorems (AT) and to explain their use with respect to model categories. These theorems are used in the literature by Mark Hovey and by Dominic Verity but not under this name. We describe their use in developing model structures on presheaf categories, in general and then for a specific presheaf category. In the first part of the talk, we describe the role of Anodyne Theorems and their relationship to the concept of homotopy in demonstrating model structures. Examples of its use can be found in the work of Hovey and Verity with respect to monoidal model categories. In the second part of the talk, we describe the specific methods used to demonstrate an Anodyne Theorem on the category of Θ_2-sets. First though we describe the AT for the model structure on simplicial sets whose fibrant objects are quasi-categories. We then describe the Θ_2-sets and lift the AT for simplicial sets to the context of Θ_2-sets. This requires the use of Day Convolution to define a pushout product of n variables (akin to the pushout product of 2 variables.) The corner tensor is an essential piece of the AT in this context and we provide a sketch of its construction.

4:30-4:50: Enxin Wu

A homotopy theory for diffeological spaces

Manifolds are favorite objects in mathematics. However, the category of manifolds are not so pleasant, for example, not every subset or quotient set of a manifold is again a manifold, and there is no standard way to talk about infinite dimensional manifolds which appear all the time, for instance, the loop space or the diffeomorphism group of a manifold, etc. Over the years, people are looking for nicer categories which contains the category of manifolds as a full subcategory, and on which we can still do differential geometry. There are many such kinds of generalizations, and one of them is called diffeological spaces, which was introduced by J. Souriau and further developed by P. Iglesias-Zemmour. One of the most beautiful thing in that theory is the irrational torus, on which the smooth homotopy groups differ from the usual continuous homotopy groups. We are trying to develop a homotopy theory on the category of diffeological spaces which extends the usual homotopy theory of manifolds and respects the smooth homotopy of the irrational torus. I will discuss some nice consequences from this theory. This is work in progress with my supervisor Dan Christensen.

5-5:20PM: Shaun Ault

Elements Partially Annihilated by the Steenrod Algebra

Let V_s be the elementary Abelian group of rank s, and let $\widetilde{\Gamma}_{s,*}$ be the reduced homology of BV_s with coefficients in $\mathbb{F}_2$, for $s \geq 1$, and set $\widetilde{\Gamma}_{0,*} = H_*(*)$. The bigraded $\mathbb{F}_2$-space, $\widetilde{\Gamma} = \{\widetilde{\Gamma}_{s,*}\}_{s\geq 0}$, admits a bigraded algebra structure induced by the product $B(\mathbb{Z}/2)^{\wedge s} \times B(\mathbb{Z}/2)^{\wedge s'} \to B(\mathbb{Z}/2)^{\wedge(s+s')}$. Let $\widetilde{\Gamma}^{\mathcal{A}^+}$ be the subspace of elements of $\widetilde{\Gamma}$ that are annihilated by every positive Steenrod operation. Some time ago, David Anick showed that $\widetilde{\Gamma}^{\mathcal{A}^+}$ is a free associative algebra. Let $\Delta(k) \subset \widetilde{\Gamma}$ be the intersection of the kernels of the Steenrod operations, $Sq^1, Sq^2, Sq^4, \ldots, Sq^{2^k}$. Recent work (joint with William Singer at Fordham University) found that the subalgebras $\Delta(k)$ are also free. Further work shows a remarkable close relationship between $\Delta(k)$ and the intersection of images of the Steenrod operations $Sq^1, Sq^3, Sq^7, \ldots, Sq^{2^{k+1}-1}$. This talk highlights these results.

5:30 - 5:50PM: Ron Umble

Non-operadic Operations on Loop Cohomology

We construct a space X whose (base pointed) loop cohomology $H = H^*(\Omega X; Z_2)$ comes equipped with a nontrivial operation $\omega : H \otimes H \to H \otimes H$. This is the first known example of an induced non-operadic operation on loop cohomology.