

COURSE ANNOUNCEMENT and SYLLABUS
Mathematics 5756 (Autumn 2024)
Mathematics 5757 (Spring 2025)

Course Name: Modern Mathematical Methods in Relativity Theory I, II (a.k.a. “Applied Differential Geometry”)

Class Number: 35368 (undergrad), 35367 (graduate)

Time and Place: Autumn 2024: MWF 12:40pm; TBA.

Office Hours: Usually right after class

Credits: 3 per semester

Prerequisites:

(i) Multivariable differential calculus and linear algebra (e.g. Math 2568 and/or 5101, 5522H).

(ii) By the 4th week one should have reviewed the concept of “*the differential of a function*” so as to have grasped the relation to the “*directional derivative of that function*”.

(iii) A physics course (e.g. Physics 1250 or higher).

(iv) No prior knowledge of tensor calculus is assumed. However, we do assume a mature attitude towards mathematics and physics.

Audience: Mature undergraduate and graduate

Textbooks:

(i) *Gravitation* by C. W. Misner, K. S. Thorne, and J. A. Wheeler (see above).

(ii) Selections from *Spacetime Physics*, 2nd edition, by E. Taylor and J.A. Wheeler

(iii) Selections from *Lecture Notes on Elementary Topology and Geometry* by I. M. Singer.

(iv) Selections from *Mathematical Methods of Classical Mechanics* by V. I. Arnold.

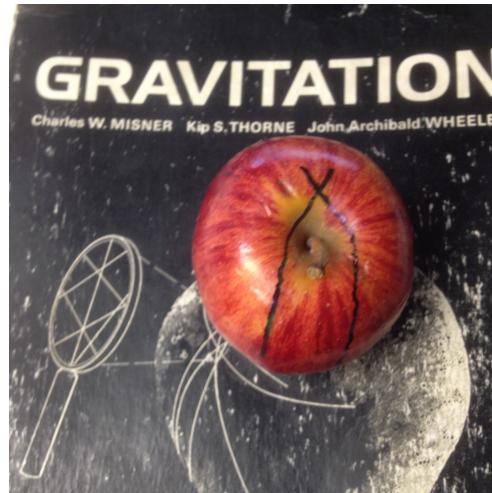
Note: (i)-(iii) exist as PDFs on the internet.

Purpose, Goal and Benefits:

1. Develop *from the bottom up* the fundamental mathematical concepts and methods responsible for the successes in 20th century physics, mathematics, and theoretical engineering.

Thus Math 5756 concretizes these developments in terms of

- (a) Special Relativity as the cognitive bridge to 20th century geometry,
- (b) multilinear algebra as a source of geometrical structures,



- (c) linear algebra's marriage to multi-variable calculus,
 - (d) differential geometry as a three level hierarchy characterized by its
 - Differential structure,
 - Parallel transport structure (a.k.a. covariant derivative)
 - Metric structure.
 - (e) the exterior calculus
 - (f) Cartan's two structural equations and their application to the Cartan-Misner calculus.
2. From the mathematical perspective these developments serve as a prelude to global mathematics.
 3. From the philosophic perspective these developments (and those in Math 5757) serve to illustrate that "Mathematics Is About the World".

Agenda:

a) Assimilate the mathematical chapters of our primary text ("*Gravitation*" by MTW, see references above), thus to develop an appreciation and the modern machinery for the mathematical framework of the laws of physics from the spacetime perspective. The development will focus on

- (1) the underlying differential geometric framework of spacetime, and
- (2) the formulation (arising from classical mechanics, fluid dynamics, and wave mechanics) of its properties.

b) Show *why* and how mathematics is the language of physical science, in particular of those aspects of physics dealing with processes of extreme violence (relativistic hydrodynamics, relativistic laser-matter interaction, high energy density physics, gravitational collapse in flat or curved spacetimes).

Exams:

All exams are in the form of *individual* open-book open-notes take-home exams.

Homework:

There will be about eight homework sets to be submitted in the form of Latex-based PDF's.

Website:

<https://people.math.osu.edu/gerlach.1/math5756>

DESCRIPTION

- Math 5756 (Autumn):
- A rapid course in special relativity: spacetime geometry, event horizons and accelerated frames;
 - tensors, metric geometry vs symplectic geometry;
 - exterior calculus, Maxwell field equations;
 - manifolds, Lie derivatives, and Hamiltonian dynamics in phase space;
 - parallel transport, torsion, tensor calculus;

- curvature and Jacobi's equation of geodesic deviation;
- Cartan's two structural equations, metric induced properties, and Cartan-Misner curvature calculus.

Math 5757 (Spring):

- The Newtonian basis of Einstein's geometrical perspective;
- Geodesics: Hamilton-Jacobi theory, the principle of constructive interference;
- stress-energy tensor: hydrodynamics in curved spacetime;
- Einstein field equations: moment of rotation = momentum-energy;
- The conservation laws and the Bianchi identities mathematized in terms of the "Boundary of a Boundary is zero ($\partial \partial \Omega = 0$)" Principle.
- Solutions to the Einstein's field equations: stars, black holes, gravitational collapse, geometry and dynamics of the universe;
- vector harmonics, tensor harmonics, acoustic and gravitational waves in violent relativistic backgrounds.

I am glad to answer any questions.

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