MINIMAL POSITIVE SEMIDEFINITE RANK OF MATRICES ASSOCIATED WITH BIPARTITE GRAPHS

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Abstract of Talk: Given a simple undirected graph G with n vertices labeled by $\{1, 2, .., n\}$ we associate a Hermitian nxn matrix A whose ith column (resp. row) represents the ith vertex and the (i,j) entry is nonzero if and only if vertex i and j are connected. The diagonal entries can be arbitrarily assigned. Let M(G) be the set of all such A and let K(G) be the subset of M(G) whose elements are positive semi-definite(PSD), then the minimum PSD rank of G, or msr(G) $= min\{rank(B) : B \in K(G)\}$. In this talk, we study the minimum PSD rank of bipartite graphs, based on a result on the rank of certain block matrices. First, we apply Lagrange multiplier method to give an algorithm telling whether a given bipartite graph has rank minm,n, where m and n denote the cardinality of the two partites of the graph. Next, we conjecture a necessary and sufficient condition for the submatrix representing the edges between the two partites to be orthonormalizable, i.e., the columns of the submatrix are non-degenerate and can be made orthogonal to one another. This condition turns out to be equivalent to msr(G) $= min\{m, n\}$, which is the absolute minimum rank such graphs can attain. Without the proof of the general conjecture, we proceed with large classes of matrices satisfying the condition of the conjecture. We give a highly involved proof that any "refinement" of a Hessenberg-like matrix pattern is orthonormalizable. We also generalize that proof to include a much larger class of matrices and their refinement. We use the local submersion theorem to give criterion for an arbitrary row extension of a matrix to be orthonormalizable, and finally we give some purely analytic proofs for certain refinement of so-called n-layered zero-diagonal matrix patterns to be orthonormalizable. Other than minimum psd rank application, the problem of finding orthonormalizable matrix patterns has a strong geometric appeal as well.