Factorization and Delta Sets in $K[x;S]$

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Abstract of Talk: In a semigroup ring $K[x;S]$, where K is a field and S is a numerical monoid, a polynomial $f(x)$, may not factor uniquely. Let $A(K[x;S])$ represent the set of irreducible elements of $K[x;S]$. Define the set of lengths of $f(x)$ by

$$\mathcal{L}(f(x)) = \{m \mid m \in \mathbb{N} \text{ such that there exist } g_1, \ldots, g_m \in A(K[x;S]) \text{ with } f(x) = g_1 \ldots g_m\}.$$

Order $\mathcal{L}(f(x)) = \{l_1, \ldots l_t\}$ with $l_{i+1} \geq l_i$ for $1 \leq i < t$. We define the Delta set of f(x) by $\Delta(f(x)) = \{l_{i+1} - l_i \mid 1 \leq i < t\}$ and the Delta set of $K[x;S]$ by

$$\Delta(K[x;S]) = \bigcup_{f(x) \in K[x;S]} \Delta(f(x)).$$

There are very few examples where the delta set of a monoid is known. We have that when K finite and $S = \langle n, n+1 \ldots, 2n-1 \rangle$, $\Delta(K[x;S]) = \Delta(G_s)$ where G_s is an abelian group dependant on both K and S.