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Coin Weighing Problems

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Abstract of Talk: Among a set of n coins of two weights (good and bad), and using a balance, we wish to determine the number of bad coins using as few measurements as possible. We are also interested in how many measurements it takes to determine the parity of this number. There are two versions of these questions, one in which we must write down all measurements to be performed beforehand (an oblivious decision tree), and one in which we can use the information gleaned from previous measurements to choose the next one (an adaptive decision tree).

There is a known adaptive decision tree that answers this question in $O((\log(n))^2)$ measurements, and a slight modification of this decision tree determines the parity of the number of bad coins in $O(\log n)$ measurements. In this talk, I will show an $\Omega(\sqrt{n})$ lower bound on the depth of any oblivious decision tree which solves either the counting or the parity problem. This demonstrates an exponential gap between the nonadaptive and adaptive decision tree complexities of these problems. It also implies the tight order of magnitude $\Theta(\log n)$ for the adaptive decision tree complexity of the parity problem.