

REVIEW PROBLEMS - SOLUTIONS

READ THIS NOTE: I will be using parenthesis "(,)" and brackets "[,]" interchangeably (when there are too many parenthesis involved, I will put brackets to clear the situation a bit out, so you can see where one begins and where one ends an expression).

Also, I will be using exclusively the notation y' , $f'(x)$, $h'(z)$ etc for the derivative. This doesn't, certainly, mean that notations such as $\frac{dy}{dx}$, $\frac{df}{dx}$ etc are not used, or invalid. If you prefer using the latter notation, kindly replace, without any penalty, accordingly: y' with $\frac{dy}{dx}$, $f'(x)$ with $\frac{df}{dx}$, etc.

Any comments or corrections regarding these solutions should be immediatly directed to me:

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Good luck!

- (1) 13/p.608 $y = (x^2 + 6x)(x^3 - 6x^2 + 4)$ - it's a product, hence use **product rule**:

$$\begin{aligned}y' &= (x^2 + 6x)' \cdot (x^3 - 6x^2 + 4) + (x^2 + 6x) \cdot (x^3 - 6x^2 + 4)' = \\ &= (2x + 6) \cdot (x^3 - 6x^2 + 4) + (x^2 + 6x) \cdot (3x^2 - 6 \cdot 2x)\end{aligned}$$

- (2) 15/p.608 $f(x) = (2x^2 + 4x)^{100}$ - it's a function to the power 100; use **chain rule** or the **power rule shortcut**:

$$f'(x) = 100(2x^2 + 4x)^{100-1} \cdot (2x^2 + 4x)' = 100(2x^2 + 4x)^{99} \cdot (2 \cdot 2x + 4)$$

- (3) 17/p.608 $y = \frac{3}{2x+1}$ - it's a fraction; use **quotient rule**:

$$\begin{aligned}y' &= \frac{3' \cdot (2x + 1) - 3 \cdot (2x + 1)'}{(2x + 1)^2} = \\ &= \frac{0 - 3 \cdot 2}{(2x + 1)^2} = \frac{-6}{(2x + 1)^2}\end{aligned}$$

- (4) 19/p.608 use product rule AND power rule (for the $(x^2 + 1)^4$ factor); get:

$$y' = 2 \cdot (x^2 + 1)^4 + (8 + 2x) \cdot 4 \cdot (x^2 + 1)^3 \cdot 2x$$

- (5) 21/p. 608 use quotient rule; get:

$$f'(z) = \frac{2z \cdot (z^2 + 4) + (z^2 - 1) \cdot 2z}{(z^2 + 4)^2}$$

- (6) 25/p.608 $y = \frac{1}{\sqrt{1-x}}$; first **rewrite**, using the fact that square root is, in fact, the power $\frac{1}{2}$:

$$y = \frac{1}{(1-x)^{\frac{1}{2}}}$$

Now you can use quotient rule. Yet, a faster way of doing it (which can be used for # 17 as well) is to keep rewriting the function, using the fact that a power in the denominator means a **negative** power in the numerator:

$$y = (1-x)^{-\frac{1}{2}}$$

Now use power rule:

$$\begin{aligned} y' &= -\frac{1}{2}(1-x)^{-\frac{1}{2}-1} \cdot (1-x)' = \\ &= -\frac{1}{2}(1-x)^{-\frac{3}{2}} \cdot (-1) = \\ &= \frac{1}{2}(1-x)^{-\frac{3}{2}} \end{aligned}$$

- (7) 27/p.608 product rule AND power rule; get:

$$h'(x) = 4(x-6)^3 \cdot 1 \cdot (x+5)^3 + (x-6)^4 \cdot 3(x+5)^2 \cdot 1$$

- (8) 29/p.608 quotient rule; get:

$$y' = \frac{5 \cdot (x+6) - (5x-4) \cdot 1}{(x+6)^2}$$

- (9) 31/p.608 CAREFUL!! first "2" is a COEFFICIENT, while the second "2" is INSIDE the power; use regular power rule for the first term, use chain rule for the second:

$$\begin{aligned} y' &= 2 \cdot (x^{-\frac{3}{8}})' + ((2x)^{-\frac{3}{8}})' = \\ &= 2 \cdot \left(-\frac{3}{8}\right)x^{-\frac{3}{8}-1} + \left(-\frac{3}{8}\right)(2x)^{-\frac{3}{8}-1} \cdot (2x)' = \\ &= 2 \cdot \left(-\frac{3}{8}\right)x^{-\frac{11}{8}} + \left(-\frac{3}{8}\right)(2x)^{-\frac{11}{8}} \cdot 2 \end{aligned}$$

- (10) 33/p.608 $y = \frac{x^2+6}{\sqrt{x^2+5}}$. Fraction - hence use quotient rule. But first, rewrite the square root as a power of $\frac{1}{2}$!

$$\begin{aligned} y &= \frac{x^2 + 6}{(x^2 + 5)^{\frac{1}{2}}} \\ y' &= \frac{(x^2 + 6)' \cdot (x^2 + 5)^{\frac{1}{2}} - (x^2 + 6) \cdot [(x^2 + 5)^{\frac{1}{2}}]'}{[(x^2 + 5)^{\frac{1}{2}}]^2} = \\ &= \frac{2x \cdot (x^2 + 5)^{\frac{1}{2}} - (x^2 + 6) \cdot \frac{1}{2}(x^2 + 5)^{\frac{1}{2}-1} \cdot (x^2 + 5)'}{(x^2 + 5)} = \\ &= \frac{2x \cdot (x^2 + 5)^{\frac{1}{2}} - (x^2 + 6) \cdot \frac{1}{2}(x^2 + 5)^{-\frac{1}{2}} \cdot 2x}{(x^2 + 5)} \end{aligned}$$

- (11) 35/p.608 $y = (x^3 + 6x^2 + 9)^{\frac{3}{5}}$. Power rule:

$$\begin{aligned} y' &= \frac{3}{5}(x^3 + 6x^2 + 9)^{\frac{3}{5}-1} \cdot (x^3 + 6x^2 + 9)' = \\ &= \frac{3}{5}(x^3 + 6x^2 + 9)^{-\frac{2}{5}} \cdot (3x^2 + 6 \cdot 2x) \end{aligned}$$

- (12) 37/p.608 $g(z) = \frac{-7z}{(z-1)^{-1}}$. This is a case of a "faux"-quotient rule - it's not a quotient, but a product, since a power of "-1" in the denominator means a power of "1" in the numerator, so we can rewrite g as follow:

$$g(z) = (-7z) \cdot (z - 1)$$

Product rule:

$$\begin{aligned} g(z) &= (-7z)' \cdot (z - 1) + (-7z) \cdot (z - 1)' = -7 \cdot (z - 1) + (-7z) \cdot 1 = \\ &= -7(z - 1) - 7z = -14z + 7 \end{aligned}$$

Note: you could've multiplied the two in fact, and avoid using the product rule - not that I recommend doing this, but if it looks simpler that way, just DO it that way!

- (13) 1/p.640 $y = 2e^x + e^2 + e^{x^2}$. Note that the second term is simply a constant. Use **exponential differentiation shortcut** (that is, the derivative is: e to the power the function TIMES derivative of power) for the third term, while the first is simply the derivative of e^x :

$$\begin{aligned}y' &= 2(e^x)' + (e^2)' + (e^{x^2})' = 2e^x + 0 + e^{x^2} \cdot (x^2)' = \\ &= 2e^x + e^{x^2} \cdot 2x\end{aligned}$$

- (14) 3/p.640 $f(r) = \ln(r^2 + 5r)$. Use **logarithms differentiation shortcut** (that is, the derivative is: 1 over function inside the ln TIMES derivative of that function):

$$f'(r) = \frac{1}{r^2 + 5r} \cdot (r^2 + 5r)' = \frac{1}{r^2 + 5r} \cdot (2r + 5)$$

- (15) 5/p.640 $y = e^{x^2+4x+5}$. Use the exponential deriv. shortcut:

$$y' = e^{x^2+4x+5} \cdot (x^2 + 4x + 5)' = e^{x^2+4x+5} \cdot (2x + 4)$$

- (16) 7/p.640 $y = e^x(x^2 + 2)$. Product, hence use product rule:

$$\begin{aligned}y' &= (e^x)' \cdot (x^2 + 2) + e^x \cdot (x^2 + 2)' = e^x \cdot (x^2 + 2) + e^x \cdot 2x = \\ &= e^x(x^2 + 2 + 2x)\end{aligned}$$

- (17) 9/p.640 $y = \sqrt{(x-6)(x+5)(9-x)}$. Since it is involving a PRODUCT of 3 factors AND a square root - which are all operation that can be SIMPLIFIED by the logarithm - we use **logarithmic differentiation**; for that, first compute the natural log of y :

$$\begin{aligned}\ln(y) &= \ln(\sqrt{(x-6)(x+5)(9-x)}) = \ln([(x-6)(x+5)(9-x)]^{\frac{1}{2}}) = \\ &= \frac{1}{2} \cdot \ln((x-6)(x+5)(9-x)) = \frac{1}{2}(\ln(x-6) + \ln(x+5) + \ln(9-x))\end{aligned}$$

(the square root is a power of $\frac{1}{2}$ applied to the WHOLE product, and hence it can be taken out of the log and set as coefficient; the product simply becomes a sum of logarithms of each factor, respectively).

Now compute the DERIVATIVE of this new function:

$$\begin{aligned}(\ln(y))' &= \left(\frac{1}{2}(\ln(x-6) + \ln(x+5) + \ln(9-x))\right)' = \\ &= \frac{1}{2}[(\ln(x-6) + \ln(x+5) + \ln(9-x))]' =\end{aligned}$$

(since $\frac{1}{2}$ is a coefficient, it can be conveniently taken out of the derivative and set as coefficient of the result)

$$\begin{aligned}&= \frac{1}{2}[(\ln(x-6))' + (\ln(x+5))' + (\ln(9-x))'] = \\ &= \frac{1}{2}\left(\frac{1}{x-6} \cdot (x-6)' + \frac{1}{x+5} \cdot (x+5)' + \frac{1}{9-x} \cdot (9-x)'\right) = \\ &= \frac{1}{2}\left(\frac{1}{x-6} + \frac{1}{x+5} + \frac{1}{9-x} \cdot (-1)\right)\end{aligned}$$

Now, by the formula of the logarithmic differentiation, we have that:

$$y' = y \cdot (\ln(y))' = \sqrt{(x-6)(x+5)(9-x)} \cdot \frac{1}{2}\left(\frac{1}{x-6} + \frac{1}{x+5} + \frac{1}{9-x} \cdot (-1)\right)$$