SOLUTIONS CHAPTER 15.1

MATH132 WI01

4. Call the length of the big rectangle L and the width W. The total length of fence needed, let's call it Y, is the rectangle's perimeter PLUS the lengths of the 3 inner fences $\rightarrow$ $(2L + 2W) + 3W = 2L + 5W$. What we need now is a relation between L and W: since the area encompased by the fence is 1000, and can be obtained using formula for area of a rectangle, namely $area = L * W \rightarrow L * W = 1000 \rightarrow W = \frac{1000}{L}$. Now we can plug this formula for W in the Y formula $\rightarrow Y = 2 * L + 5 * W = 2L + 5 * \frac{1000}{L} = 2L + \frac{5000}{L} = \frac{2L^2}{L} + \frac{5000}{L} = \frac{2L^2+5000}{L}$. We're looking for **least length** hence we need the minimum for the Y. Compute derivative of Y with respect to L (that is, L is the variable):

$$Y' = \frac{2 * 2L * L - (2L^2 + 5000)}{L^2} = \frac{4L^2 - 2L^2 - 5000}{L^2} = \frac{2L^2 - 5000}{L^2}.$$

$$Y' = \frac{2(L^2 - 2500)}{L^2} = \frac{2(L - 50)(L + 50)}{L^2}.$$

For the minimum we need critical numbers, that is zeroes for Y', so $L = \pm 50$, and also Y' =DNE, hence $L = 0$. But we have some practical reasons to ignore some of these values: the length cannot be negative, and cannot be 0 as well. So the only one that remains in 50 ... but is it a minimum? plug in 40 and 60 in Y', we get $(-)$ and $(+)$, respectively, that is the Y is $\searrow$ and then $\nearrow$ ... hence 50 IS a **minimum**. Hence $L = 50$, and $W = \frac{1000}{L} = \frac{1000}{50} = 20$, so then $Y = 2L + 5W = 2 * 50 + 5 * 20 = 100 + 100 = 200$.

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6. $C = 0.12s - 0.0012s^2 + 0.08$, and we have that $0 \leq s \leq 60 \rightarrow s \in [0.60]$. We're looking for lowest C, so we need $C' = 0.12 - 0.0012 * 2s = 0.12 - 0.0024s = 0.0024(\frac{0.12}{0.0024} - s) = 0.0024(50 - s)$. Critical number is 50, and we notice (by pluging, let's say, 45 and 55 in C', getting $(+)$ and $(-)$, meaning$\nearrow$ and $\searrow$ respectively, for C) that 50 is a maximum ... but we are looking for the **minimum**! Our last chance is to try out the endpoints (as you remember, to find absolute maximum and minimum in an interval one has to plug critical numbers **and** endpoints in the original function). Speed varies in the interval $[0, 60]$, so let's plug in 0 in C: $C(0) = 0.08$; plug in 60 in C: $C(60) = 0.12*60 - 0.0012*60^2 + 0.08 = 2.96$. Lowest value we get for $s = 0$ and this is the speed where cost is minimum (this means one should just leave its car in the garage, to minimize cost of using it ... as if!).

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18. Let's call x the multiple of 10s of rent's increase from 400 (which means that, in terms of x, the rent will be $400 + 10 * x$). The number of rented apartments will then be $100 - 2 * x$ (since for every new multiple of 10 there are **2**

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new vacant apartments). The revenue (call it R) is R=(rent)*(number of rented aparments)=$(400 + 10x) * (100 - 2x)$. To find maximum we need $R' = 10 * (100 - 2x) + (400 + 10x) * (-2) = 1000 - 20x - 800 - 20x = 200 - 40x = 40(5 - x)$. The critical number is 5, and it is a **maximum** (pluging 4 and 6 respectively we get (+) and (−), which means R is $\nearrow$ and then $\searrow$). Hence, for $x = 5$ the revenue is maximized, so the rent that produces this is $400 + 10 * x = 400 + 10 * 5 = 450$.

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22. The material used's area is (area of base)+(area of all sides)=(area of the base square) + (4 times the area of one of the sides) (remember, it's an **open-top** box; also, all sides are same, since all have x and y as length and width). Hence, since material used is 192, we have: $192 = (x^2) + 4 * (xy)$; let's find y in terms of x (why? because finding x in terms of y will be way more difficult, since we get a **quadratic** equation; as for y, it's just a linear equation, as you will see in the following argument). Hence, $192 - x^2 = 4xy \to y = \frac{192-x^2}{4x}$. Now ... the volume of the box is (length)*(width)*(height)=$x * x * y$. So, volume (call it V) will be $V = x^2 * \frac{192-x^2}{4x} = x * \frac{192-x^2}{4} = \frac{1}{4} * (192x - x^3)$. To find maximum, we need $V' = \frac{1}{4} * (192 - 3x^2) = \frac{1}{4} * 3 * (64 - x^2) = \frac{3}{4}(8 - x)(8 + x)$. Critical numbers are ± 8, but, for practical reasons (since lengths of boxes should be kept positive), we ignore -8, so we need just $x = 8$. Is it a maximum? (quite likely, but ...) let's check (plug 7 and 9 in V', we get (+) and (−), meaning $\nearrow$ and then $\searrow$ for V). Hence, if $x = 8$, then $y = \frac{192-x^2}{4x} = \frac{192-64}{4*8} = 4$, so $V = 8 * 8 * 4 = 256$.

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