

## SOLUTIONS SHEET FOR SAMPLE PROBLEMS MIDTERM I

MATH 132 WI01

I. Compute the following limits (if the limit is  $+\infty$  or  $-\infty$  or DNE, state whether it is  $+\infty$  or  $-\infty$  or DNE; leave the answer in fractions)

$$(a) \quad \lim_{x \rightarrow 2} \frac{4 - x^2}{x^2 - x - 2}$$

**Answer:**  $-\frac{4}{3}$

$$(b) \quad \lim_{x \rightarrow \infty} \frac{x^2 + 6x + 9}{(7x - 5)^2}$$

**Answer:**  $\frac{1}{49}$

$$(c) \quad \lim_{x \rightarrow 3^+} \sqrt{x - 3}$$

**Answer:** 0

$$(d) \quad \lim_{x \rightarrow -\infty} \frac{5x - 7}{x - 7}$$

**Answer:** 5

$$(e) \quad \lim_{t \rightarrow 2} \frac{7t - 4}{t - 2}$$

**Answer:** *DNE*

$$(f) \quad \lim_{h \rightarrow 0} \frac{\frac{5}{4+h} - \frac{5}{4}}{h}$$

**Answer:**  $-\frac{5}{16}$

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$$(h) \quad \lim_{x \rightarrow -5} \frac{x^2 - 25}{x - 5}$$

**Answer:** 0

$$(i) \quad \text{If } f(x) = 11x^2 + 7, \text{ find } \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Answer:**  $22x$

$$(j) \quad \lim_{x \rightarrow -\infty} \frac{4 - x^2}{1 - x}$$

**Answer:**  $-\infty$

$$(k) \quad \lim_{x \rightarrow \infty} \frac{5 - x^2}{x^4 - 8x^2 + 2}$$

**Answer:** 0

$$(l) \quad \lim_{x \rightarrow 0^-} \frac{x^2 + 4}{x}$$

**Answer:**  $-\infty$

$$(m) \quad \lim_{x \rightarrow 1^-} \frac{7}{1 - x}$$

**Answer:**  $\infty$

$$(n) \quad \lim_{x \rightarrow 1} \frac{7}{1 - x}$$

**Answer:** *DNE*

$$(o) \quad \lim_{x \rightarrow 0} \frac{7}{x}$$

**Answer:** *DNE*

$$(p) \quad \lim_{x \rightarrow 0} \frac{7}{x^2}$$

**Answer:**  $\infty$

II. Let

$$f(x) = \begin{cases} 5, & \text{if } x > 4 \\ x, & \text{if } x \leq 4 \end{cases}$$

$$(a) \quad \text{Find } \lim_{x \rightarrow 4^+} f(x)$$

**Answer:** 5

$$(b) \quad \text{Find } \lim_{x \rightarrow 4^-} f(x)$$

**Answer:** 4

$$(c) \quad \text{Find } \lim_{x \rightarrow 4} f(x)$$

**Answer:** *DNE*

$$(d) \quad \text{Find } \lim_{x \rightarrow \infty} f(x)$$

**Answer:** 5

$$(e) \quad \text{Find } \lim_{x \rightarrow -\infty} f(x)$$

**Answer:**  $-\infty$

III. Find the derivatives of the following functions (do not simplify)

$$(a) \quad f(x) = (2 + 15x - 7x^2)(x^9 - 8x + 4)$$

**Answer:**  $(15 - 14x)(x^9 - 8x + 4) + (2 + 15x - 7x^2)(9x^8 - 8)$

$$(b) \quad f(t) = \frac{t^3 + 7t + 5}{t^5 - 9t^2 + 3}$$

$$\textbf{Answer: } \frac{(3t^2+7)(t^5-9t^2+3)-(t^3+7t+5)(5t^4-18t)}{(t^5-9t^2+3)^2}$$

$$(c) \quad y = (8x + 5)^{\frac{4}{7}} + e^{\frac{4}{7}}$$

$$\textbf{Answer: } \frac{4}{7}(8x + 5)^{-\frac{3}{7}}(8)$$

$$(d) \quad p = \frac{5}{q}$$

$$\textbf{Answer: } -\frac{5}{2}q^{-\frac{3}{2}}$$

$$(e) \quad y = \frac{(3x + 1)^5 - 5x}{7}$$

$$\textbf{Answer: } \frac{1}{7}[5(3x + 1)^4(3) - 5]$$

$$(f) \quad s = (5t^3 + 1)^7(3t - 4)^6$$

$$\textbf{Answer: } [7(5t^3 + 1)^6(15t^2)](3t - 4)^6 + (5t^3 + 1)^7[6(3t - 4)^5(3)]$$

IV. Find the equation of the tangent line to the graph of  $y = x^3 - 7x + 6$  at the point  $(1, 0)$

$$\textbf{Answer: } y = -4x + 4$$

V. Find the rate of change of  $f(x) = 9x^6 - 8x^3 + 11$  with respect to  $x$  and evaluate it when  $x = 1$

$$\textbf{Answer: } 54x^5 - 24x^2; 30$$

VI. If a manufacturer's average cost function is given by

$$\bar{C} = .005q^2 - .5q + 70 + \frac{300}{q},$$

find the marginal cost function. What is the marginal cost when 50 units are produced?

$$\textbf{Answer: } .015q^2 - q + 70; 57.5$$

VII. Use *Chain Rule* to find  $\frac{dy}{dx}$  where  $y = 3u^8 - 4$  and  $u = 7x^3 + 3x - 1$

**Answer:**  $24u^7(21x^2 + 3) = 24(7x^3 + 3x - 1)^7(21x^2 + 3)$

VIII. Use *Chain Rule* to find  $\frac{dy}{dx}$  where  $y = 11u^5 - 7u + 11$  and  $u = 9x^6 - 8x^3 + 11$

**Answer:**  $(55u^4 - 7)(54x^5 - 24x^2) = [55(9x^6 - 8x^3 + 11)^4 - 7](54x^5 - 24x^2)$

IX. Solve the inequality

(a)  $\frac{(2-x)(x-5)}{11-x} > 0$

**Answer:**  $(2, 5) \cup (11, \infty)$

(b)  $\frac{(7-x)(x-9)}{8-x} \leq 0$

**Answer:**  $(-\infty, 7] \cup (8, 9]$