

Math 116—The Mathematics of Voting

Voting Methods

1. Plurality Method

The candidate with the most first-place votes wins.

Notes:

2. Borda Count Method

Each place on a ballot is assigned points. If there are N candidates, the last candidate gets 1 point, the second-to-last candidate gets 2 points, ... $N - 1$ points for second place, and N points for first place. The points are tallied for each candidate separately and the candidate with the highest total is the winner.

Notes:

3. Plurality-with-Elimination Method (Instant Runoff Voting)

- (i) Count the first place votes for each candidate. If a candidate has a majority of first-placed votes, that candidate is the winner.
- (ii) If no candidate has a majority, eliminate the candidate with the fewest first-placed votes.
- (iii) Go back to step (i).

Notes:

4. Method of Pairwise Comparisons

For each pair of candidates, determine which would win a head-to-head election; that is, each vote is assigned to whichever of the two candidates is higher on the ballot. Once we've gone through all the ballots (are until one of the two candidates has a majority), the winning candidate is assigned one point—if there's a tie, each candidate receives $\frac{1}{2}$ point.

We go through all possible pairs of candidates, and tally all the points. The candidate with the most points wins the election.

Notes:

Fairness Criteria

1. Majority Criterion

If there is a candidate that has a majority of the first-place votes, then that candidate should be the winner of the election.

Notes:

2. Condorcet Criterion

If there is a candidate that is preferred by the voters over each of the other candidates (called a Condorcet candidate), then that candidate should be the winner of the election.

Notes:

3. Monotonicity Criterion

If choice X is a winner of an election and, in a re-election, all the changes in the ballots are changes favorable only to X , then X should still be a winner of the election.

Notes:

4. Independence-of-Irrelevant-Alternatives Criterion

If choice X is a winner of an election, and one (or more) of the other choices is disqualified and the ballots recounted, then X should still be a winner of the election.

Notes:

Examples

1. The OSU marching band was invited to five different bowl games: The Rose Bowl (R), the Hula Bowl (H), the Cotton Bowl (C), the Orange Bowl (O), and the Sugar Bowl (S). An election is held among the 100 members of the band to decide which bowl game they should attend. The preference schedule is given below:

Number of voters	49	48	3
1st choice	<i>R</i>	<i>H</i>	<i>C</i>
2nd choice	<i>H</i>	<i>S</i>	<i>H</i>
3rd choice	<i>C</i>	<i>O</i>	<i>S</i>
4th choice	<i>O</i>	<i>C</i>	<i>O</i>
5th choice	<i>S</i>	<i>R</i>	<i>R</i>

2. The principal of Centennial High has retired and the school board must hire a new principal. The four finalists for the job are Mrs. Amaro, Mr. Burr, Mr. Castro, and Mrs. Dunbar (A,B,C, and D, respectively). After interviewing the four finalists, the eleven members of the school board vote by ranking the four candidates. The preference schedule is given below:

Number of voters	6	2	3
1st choice	<i>A</i>	<i>B</i>	<i>C</i>
2nd choice	<i>B</i>	<i>C</i>	<i>D</i>
3rd choice	<i>C</i>	<i>D</i>	<i>B</i>
4th choice	<i>D</i>	<i>A</i>	<i>A</i>

3. The preference schedule below is for an election between five candidates, A, B, C, D, and E:

Number of voters	10	5	2	1	4	4
1st choice	<i>A</i>	<i>B</i>	<i>C</i>	<i>C</i>	<i>D</i>	<i>E</i>
2nd choice	<i>B</i>	<i>D</i>	<i>A</i>	<i>E</i>	<i>C</i>	<i>D</i>
3rd choice	<i>C</i>	<i>E</i>	<i>E</i>	<i>B</i>	<i>A</i>	<i>C</i>
4th choice	<i>D</i>	<i>C</i>	<i>B</i>	<i>A</i>	<i>E</i>	<i>A</i>
5th choice	<i>E</i>	<i>A</i>	<i>D</i>	<i>D</i>	<i>B</i>	<i>B</i>

4. Three cities, Rome (R), Buenos Aires (B), and Chicago (C), are competing to hold the next Summer Olympic Games. The final decision is made by a secret vote of the 29 members of the Executive Council of the International Olympic Committee. Suppose a straw vote is conducted by the Executive Council just to see how things stand, with the results given in the preference schedule below:

Number of voters	7	8	10	4
1st choice	<i>A</i>	<i>B</i>	<i>C</i>	<i>A</i>
2nd choice	<i>B</i>	<i>C</i>	<i>A</i>	<i>C</i>
3rd choice	<i>C</i>	<i>A</i>	<i>B</i>	<i>B</i>

5. San Diego had the number-one pick in the 2004 draft. They narrowed their list to five players: Eli Manning (E), Robert Gallery (R), Larry Fitzgerald (L), Philip Rivers (P), and Sean Taylor (S). The coaches and team executives meet and vote on which player to draft. The preference schedule for their votes is given below:

Number of voters	2	6	4	1	1	4	4
1st choice	<i>E</i>	<i>R</i>	<i>R</i>	<i>L</i>	<i>L</i>	<i>P</i>	<i>S</i>
2nd choice	<i>P</i>	<i>E</i>	<i>E</i>	<i>R</i>	<i>P</i>	<i>E</i>	<i>L</i>
3rd choice	<i>L</i>	<i>L</i>	<i>P</i>	<i>E</i>	<i>E</i>	<i>S</i>	<i>P</i>
4th choice	<i>R</i>	<i>P</i>	<i>S</i>	<i>P</i>	<i>R</i>	<i>L</i>	<i>R</i>
5th choice	<i>S</i>	<i>S</i>	<i>L</i>	<i>S</i>	<i>S</i>	<i>R</i>	<i>E</i>