

Apportionment Theory

Apportionment Methods

I. Hamilton's Method

1. Calculate each state's standard quota.
2. Give to each state (for the time being) its *lower quota*. In other words, round each state's quota down.
3. Give the extra seats (one at a time) to the states with the largest fractional parts until there are no more extra seats.

Notes:

II. Jefferson's Method

1. Find a modified divisor D such that when each state's *modified quota* (state's population divided by D) is rounded *downward* (*modified lower quota*), the total is the exact number of seats to be apportioned.
2. Apportion to each state its modified lower quota.

Notes:

III. Adam's Method

1. Find a modified divisor D such that when each state's *modified quota* (state's population divided by D) is rounded *upward* (*modified upper quota*), the total is the exact number of seats to be apportioned.
2. Apportion to each state its modified upper quota.

Notes:

IV. Webster's Method

1. Find a modified divisor D such that when each state's *modified quota* (state's population divided by D) is rounded *the conventional way* (to the nearest integer), the total is the exact number of seats to be apportioned.
2. Apportion to each state its modified quota rounded the conventional way.

Notes:

Apportionment Paradoxes

- I. **The Alabama Paradox** - An increase in the total number of seats being apportioned, in and of itself, forces a state to lose one of its seats.

Notes:

- II. **The Population Paradox** - Even though state X's population grew at a higher rate than state Y's did, X loses a seat to Y.

Notes:

- III. **The New-States Paradox** - The addition of a new state with its fair share of seats can, in and of itself, affect the apportionments of other states.

Notes:

The Quota Rule

- A state's fair apportionment should be either its upper quota or its lower quota.
- An apportionment *method* that guarantees that every state will be apportioned either its lower quota or its upper quota is said to *satisfy the quota rule*.
- Violations of the quota rule can be of two types:
 - **Lower-Quota Violation** - When a state ends up with an apportionment smaller than its lower quota.
 - **Upper-Quota Violation** - When a state ends up with an apportionment larger than its upper quota.

Notes:

Terms

Standard Divisor (SD) - The total population divided by the total number of seats.

State X's Standard Quota - State X's population divided by the Standard Divisor.

Lower Quota - The Standard Quota rounded down.

Upper Quota - The Standard Quota rounded up.

Balinski and Young's Impossibility Theorem - There cannot be a perfect apportionment method. Any apportionment method that does not violate the Quota Rule must produce paradoxes, and any apportionment method that does not produce paradoxes must violate the Quota Rule.

Notes:

Examples

Table 4-1

Child	Alan	Betty	Connie	Doug	Ellie	Total
Minutes Worked	150	78	173	204	295	900

Notes:

Table 4-3

State	A	B	C	D	E	F	Total
Population	1,646,000	6,936,000	154,000	2,091,000	685,000	988,000	12,500,000

Notes:

Table 4-7

State	A	B	C	Total
Population	940	9,030	10,030	20,000

Notes:

Table 4-14

School	North High	South High	Total
Enrollment	1,045	8,955	10,000

Table 4-15

School	North High	South High	New High	Total
Enrollment	1,045	8,955	525	10,525

Notes: