

Fibonacci Fun and Games

The numbers below are the beginning of the Fibonacci sequence, named for the Italian mathematician Leonardo Fibonacci (c. 1175 - 1250).

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

1. First things first, let's find the pattern. What are the next three numbers in the sequence?

2. Let's denote the N th Fibonacci number by the symbol F_N ; so for example we have $F_1 = 1$, $F_4 = 3$, $F_8 = 21$, and so on. The numbers in the sequence obey a rule that lets us determine the N th term; what is the rule?

$$F_N = \quad \quad \quad \text{for } N \geq \underline{\quad}$$

3. Just knowing the rule in the previous part is not enough to define the sequence. In addition to the rule, what else do we need to know to fully define the sequence? Write a concise definition of the Fibonacci sequence. [This is called a *recursive* definition]

4. Below are given some new sequences that are similar to the Fibonacci sequence. For each sequence, determine its rule and give a concise definition for the sequence, as you did for Fibonacci above (use a_N for the N th term of each sequence; we'll save F_N for the original Fibonacci sequence).

(a) 1, 3, 4, 7, 11, 18, 29, ...

(b) 2, 4, 6, 10, 16, 26, 42, ...

(c) 1, 1, 3, 7, 17, 41, 99, ...

(d) 1, 1, 3, 5, 11, 21, 43, 85, ...

(e) 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, ...

5. Take the sequence in part (a) of the last question: 1, 3, 4, 7, 11, 18, 29, If we multiply every term by 2, we get a new sequence: 2, 6, 8, 14, 22, What is the rule governing this new sequence? What would happen if we multiplied every term by 3 instead?
6. Let's create a new sequence by adding the terms of the sequences in parts (a) and (b) above: 3, 7, 10, 17, What can we say about this new sequence?
7. From the last two questions, we have some experimental evidence that might lead us to creating a hypothesis (or hypotheses). Formulate such a hypothesis (i.e. figure out how to say it correctly) and try to develop an argument to support it (or find a counterexample that disproves it).
8. Let's define a new sequence: $G_1 = a, G_2 = b$ and for $N \geq 3$ we have $G_N = G_{N-1} + G_{N-2}$. Write out the first several terms of this sequence. What do you notice?