DISCRETE DISTRIBUTIONS

- Uniform Distribution
- Bernoulli Trial: $\Omega = \{\text{success}, \text{failure}\}$. $P(\text{success}) = p$, $P(\text{failure}) = q = 1 - p$.
- Bernoulli Distribution $B = B(1, p)$: $P(B = 1) = p, \quad P(B = 0) = 1 - p = q$.
- Binomial Distribution $B(n, p)$: Repeat Bernoulli trial independently for n times, and let $B(n, p)$ be the number of successes. Then

$$P[B(n,p) = x] = \binom{n}{x} p^x (1-p)^{n-x}.$$

- Geometric Distribution: In the repeated trials,
  - Let X be the number of failures until the first success. Then $P(X = x) = (1 - p)^x p$
  - Let Y be the number of trials on which the first success occurs. Then $P(Y = y) = (1 - p)^{y-1} p$
- Negative Binomial Distribution: Let X be the number of failures until the rth success. Then

$$P(X = x) = \binom{r+x-1}{r-1} p^r (1-p)^x = \binom{r+x-1}{x} p^r (1-p)^x.$$

- Poisson Distribution: $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$.
- Hypergeomatric Distribution: Among total M objects, K are good and $M - K$ are bad. A sample of n objects is selected without replacement. Let X be the number of good ones in the sample. Then

$$P(X = x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}.$$

- Multinomial Distribution: An experiment has k outcomes with probabilities p_1, p_2, ... , p_k respectively. Repeat the experiment n times independently. Let X_i be the number of times that the ith outcome occurs. Then

$$P(X_1 = x_1, X_2 = x_2, ..., X_k = x_k) = \frac{n!}{x_1! \cdots x_k!} \cdot p_1^{x_1} \cdots p_k^{x_k}.$$