

1. Let  $S = \{\omega_1, \omega_2, \omega_3\}$ ,  $P(\omega_1) = P(\omega_2) = P(\omega_3) = 1/3$ . Define random variables  $X$  and  $Y$  by

$$\begin{aligned} X(\omega_1) &= 1, & X(\omega_2) &= 2, & X(\omega_3) &= 3; \\ Y(\omega_1) &= 2, & Y(\omega_2) &= 3, & Y(\omega_3) &= 1. \end{aligned}$$

Let  $p_X(x)$ ,  $p_Y(x)$ , and  $p_{X+Y}(x)$  be the probability distribution functions of  $X$ ,  $Y$ , and  $X + Y$  respectively. Fill in the following blanks:

$$p_X(0) = \underline{\quad}, \quad p_X(1) = \underline{\quad}, \quad p_X(2) = \underline{\quad}, \quad p_X(3) = \underline{\quad}, \quad p_X(4) = \underline{\quad};$$

$$p_Y(0) = \underline{\quad}, \quad p_Y(1) = \underline{\quad}, \quad p_Y(2) = \underline{\quad}, \quad p_Y(3) = \underline{\quad}, \quad p_Y(4) = \underline{\quad}.$$

$$p_{X+Y}(1) = \underline{\quad}, \quad p_{X+Y}(2) = \underline{\quad}, \quad p_{X+Y}(3) = \underline{\quad},$$

$$p_{X+Y}(4) = \underline{\quad}, \quad p_{X+Y}(5) = \underline{\quad}, \quad p_{X+Y}(6) = \underline{\quad}.$$

2. Telegraphic signals “dot” and “dash” are sent in the proportion 3:4. Due to conditions causing very erratic transmission, a dot becomes a dash with probability  $1/4$ , whereas a dash becomes a dot with probability  $1/3$ . If a dot is received, what is the probability that it was sent as a dot?

Answer \_\_\_\_\_