Math 366 Practice Problems for Exam 1 **Spring 2009**

1. Each of the following statements has one of the forms

$$\sim p \quad p \wedge q \quad p \vee q \quad p \rightarrow q \quad p \leftrightarrow q$$

Find the appropriate form and indicate what each statement variable in your choice represents.
(a) If Archibald passes the first exam, then he will not drop the course.
(b) The moon is not made of green cheese.
(c) Clem is happy if and only if he is laying in the sun.
(d) Either the rain will stop or the river will overflow.
(e) Spiro won and Frank didn't win.

2. Use a truth table to verify the following logical equivalence.
$\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

3. Show that each of the following arguments has a valid argument form by exhibiting such a form. Explain what each statement variable in your form represents. Hint: Use table 1.3.1.

(a) I'll either get a Christmas bonus or I'll sell my motorcycle.
If I get a Christmas bonus, then I'll buy a CD player.
If I sell my motorcycle, then I'll buy a CD player.
Therefore, I'll buy a CD player.

(b) If Christine intends to go to the party, then John will also.
John is not intending to go to the party.
Therefore, Christine is not intending to go to the party.

4. Use a truth table to determine whether the following argument form is valid.

$p \rightarrow (q \vee r)$

$q \rightarrow (\sim p \vee r)$

$\therefore\ p \rightarrow r$

5. Derive each argument form below using the valid argument forms in table 1.3.1 of the text and the logical equivalences in table 1.1.1 of the text. Number each step and supply a reason for each step.

(a) $p \rightarrow (q \rightarrow r)$
$\sim r$
p
$\therefore\ \sim q$

(b) $p \vee \sim q$
q
$\therefore\ p$

6. Make up an I/O table for three inputs P, Q, and R. Find a Boolean expression for your I/O table.

7. Build a circuit with inputs P, Q, and R using AND-gates OR-gates and NOT-gates.
(a) Find the output of the circuit with inputs $P = Q = 0$ and $R = 1$.
(b) Fint the I/O table of the circuit.
(c) Find a Boolean expression which is equivalent to the circuit.
Show your work.

8.Find an equivalent circuit for the following Boolean expression.
$\sim ((P \wedge Q) \vee (\sim Q \wedge R))$

9. Each of the expressions below has one of the forms

$$\forall x \in D, P(x) \qquad \exists x \in D \ s.t. \ P(x)$$

Determine the appropriate form and indicate the interpretation of the domain D and the predicate $P(x)$.
(a) Everyone in the class who works hard will pass.
(b) Someone is sleeping.

10. Find a counterexample for the following universal statement.
$\forall x \in \mathbf{R}, x^3 \neq -x$

11. Show that each of the following arguments has a valid form in predicate logic by exhibiting such a form. Justify that your form is valid. Also indicate how to interpret any domain symbols or predicate symbols you use as well as any symbol used as a name.

(a) Every math 366 exam is simple.
This exam isn't simple.
Therefore, this exam is not a math 366 exam.

(b) Every math 366 exam is simple.
This exam is a math 366 exam.
Therefore, this exam is simple.

12. Give constructive proofs of the following statements.
(a) There exists an integer x such that $x^2 < x + 1$.
(b) There exist integers x and y such that $x^2 + xy + y^2 = 1$.
(c) There is a rational number p such that $9p^2 = 4$.

13. Give the first sentence of a direct proof of the following statements. Also indicate what remains to be proved.
(a) Every integer is either even or odd.
(b) The product of two rational numbers is rational.
(c) If an integer is prime and different from 2 then it is odd.

14. Give proofs of the following.
(a) For all real numbers x, y and z, if $2x + 1 = z$ then $2(x + y) + 1 = 2y + z$.
(b) The sum of an even and an odd integer is odd.
(c) For all integers n and m, if $n - m$ is even then $n^2 - m^2$ is even.

(d) For all integer n, if n is even then $\frac{n^2}{4}$ is an integer.
(e) For every integer n, if n is even then $3n$ is divisible by 6.
(f) The reciprocal of a nonzero rational number is rational.
(g) The ratio of nonzero rational numbers is rational.