Theorem – Logical Equivalences

Given any statement variables p, q, and r, a tautology **t** and a contradiction **c**, the following logical equivalences hold.

1. *Commutative laws*:

$$p \wedge q \equiv q \wedge p \qquad\qquad p \vee q \equiv q \vee p$$

2. *Associative laws*:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \qquad\qquad (p \vee q) \vee r \equiv p \vee (q \vee r)$$

3. *Distributive laws*:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \qquad\qquad p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

4. *Identity laws*:

$$p \wedge \mathbf{t} \equiv p \qquad\qquad p \vee \mathbf{c} \equiv p$$

5. *Negation laws*:

$$p \vee \sim p \equiv \mathbf{t} \qquad\qquad p \wedge \sim p \equiv \mathbf{c}$$

6. *Double Negative law*:

$$\sim (\sim p) \equiv p$$

7. *Idempotent laws*:

$$p \wedge p \equiv p \qquad\qquad p \vee p \equiv p$$

8. *Universal bound laws*:

$$p \vee \mathbf{t} \equiv \mathbf{t} \qquad\qquad p \wedge \mathbf{c} \equiv \mathbf{c}$$

9. *De Morgan's Laws*:

$$\sim (p \wedge q) \equiv \; \sim p \vee \sim q \qquad\qquad \sim (p \vee q) \equiv \; \sim p \wedge \sim q$$

10. *Absorption laws*:

$$p \vee (p \wedge q) \equiv p \qquad\qquad p \wedge (p \vee q) \equiv p$$

11. *Negations of* **t** *and* **c**:

$$\sim \mathbf{t} \equiv \mathbf{c} \qquad\qquad \sim \mathbf{c} \equiv \mathbf{t}$$

Summary of Rules of Inference – Valid Argument Forms

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a**. $p \vee q$ $\sim q$ $\therefore p$ **b**. $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a**. p $\therefore p \vee q$ **b**. q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a**. $p \wedge q$ $\therefore p$ **b**. $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to \mathbf{c}$ $\therefore p$