Math 512: FINAL EXAMINATION

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NAME:

1. Solve the 2D boundary value heat problem in a unit square ($a = b = 1$) with the initial temperature distribution $f(x, y)$:

$$u_t = u_{xx} + u_{yy}, \qquad 0 \leqslant x \leqslant 1, \quad 0 \leqslant y \leqslant 1, \quad t \geqslant 0$$

$$u(0, y, t) = u(1, y, t) = u(x, 0, t) = u(x, 1, t) = 0,$$

$$u(x, y, 0) = f(x, y) = 1 \ .$$

2. Solve the Dirichlet problem

$$u_{xx} + u_{yy} = 0, \qquad 0 \leqslant x \leqslant 1, \quad 0 \leqslant y \leqslant 1$$

$$u(x,0,t) = f_1(x) = \sin(\pi x)\cos(\pi x), \qquad u(x,1,t) = f_2(x) = 0,$$

$$u(0,y,t) = g_1(y) = \sin^2(\pi x) - \cos^2(\pi x), \qquad u(1,y,t) = g_2(y) = 0\ .$$

3. Using the convolution theorem $\mathscr{L}(f * g) = \mathscr{L}(f) \cdot \mathscr{L}(g)$ find the inverse Laplace transform

$$\mathscr{L}^{-1}\left(\frac{s}{(s^2+16)^2}\right) .$$

4. Solve the differential equation

$$y'' + y' - 6y = 1, \qquad\qquad y(0) = 1, \quad y'(0) = 1 \ .$$

5. Use the Laplace transform to solve the following boundary value problem

$$
\begin{aligned}
&u_{tt} = u_{xx} - 2, \qquad && x \geqslant 0, \quad t \geqslant 0 \\
&u(0,t) = 0, && \\
&u(x,0) = 0, \quad u_t(x,0) = 0\ . &&
\end{aligned}
$$