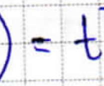
$$J_L(A) = (-A)^{-3w(L)} [L](A, A^{-1}, -A^{2}-A^{-2})$$

$$A = t^{-1/4}, \quad J($$  $$) = t^{-1}+t^{-3}-t^{-4}$$

Thistlethwaite's theorem

$$J_L(t) = \pm \underset{e(+)}{t^{N}} \, \overline{T}_G(-t, -t^{-1})$$

$$[L](A,B,d) := \sum_s A^{\alpha(s)} B^{\beta(s)} d^{\delta(s)-1}$$

$$T_G(X,Y) := \sum_{F \subseteq G} \Big(\prod_{e \in F} x_e\Big)\Big(\prod_{e \notin F} y_e\Big) X^{k(F)-k(G)} Y^{n(F)}$$

$$\overline{T}(x,y) = T_G(x-1, y-1) \quad X = x-1, \ Y = y-1$$

$k(F) :=$ # conn. comp. $\quad n(F) := e(F) - v(F) + k(F)$

Theorem

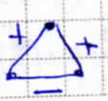
$$[L](A,B,d) = A^{n} B^{e-n} d^{k-1} \, T_G\left(\frac{Ad}{B}, \frac{Bd}{A}\right)$$

$$x_+ = y_+ = 1, \quad x_- = \cancel{B/A}\ A/B, \quad y_- = \cancel{A/B}\ B/A$$

$$S_0 \longleftrightarrow \{F = \emptyset\}$$

S_0 

$$s \longleftrightarrow F_s$$



$$F_s : A^{n(G)} B^{e(G)-n(G)} d^{k(G)-1} \Big(\prod_{e \in F_s} x_e\Big)\Big(\prod_{e \notin F_s} y_e\Big)$$

$$X^{k(F_s)-k(G)} \; Y^{n(F_s)}$$

complement

$$= A^{n(G)} B^{e(G)-n(G)} d^{k(G)-1} \left(\frac{A}{B}\right)^{e_-(F_s)} \left(\frac{B}{A}\right)^{e_-(\bar{F}_s)}$$

$$\left(\frac{Ad}{B}\right)^{k(F_s)-k(G)} \left(\frac{Bd}{A}\right)^{n(F_s)}$$

exponent of d

$$\cancel{k(G)} - 1 + k(F_s) - \cancel{k(G)} + n(F_s) =$$

$$= -1 + k(F_s) + e(F_s) - v + k(F_s)$$

$$= -1 \underbrace{- v + 2k(F_s) + e(F_s)}_{\delta(s)}$$

exponent of A

$$\underbrace{n(G)}_{e(G)-\cancel{v}} + e_-(F_s) - e_-(\bar{F}_s) + \underbrace{k(F_s) - k(G) - n(F_s)}_{-e(F_s)+\cancel{v}}$$

$$= \underbrace{e(G) - e(F_s)}_{\substack{e(\bar{F}_s) \\ \| \\ \alpha_{\bar{F}}(s) + \beta_{\bar{F}}(s)}} + \underbrace{e_-(F_s)}_{\alpha_F(s)} - \underbrace{e_-(\bar{F}_s)}_{\cancel{\beta_F(s)}} = \alpha_{\bar{F}}(s) + \alpha_F(s) = \alpha(s)$$

exponent of B

$$\underbrace{e(G) - n(G)}_{\cancel{v - k(G)}} - e_-(F_s) + e_-(\overline{F_s}) \underbrace{- k(F_s)}_{e(F_s) - \cancel{v}} + \cancel{k(G)} + n(F_s)$$

$$= e(F_s) - e_-(F_s) + e_-(\overline{F_s})$$

$$= \underbrace{e_+(F_s)}_{\beta_F(s)} + \underbrace{e_-(\overline{F_s})}_{\beta_{\overline{F}}(s)} = \beta(s)$$