Partial duality of hypermaps

Lyon, May 27, 2014

3D PLAN

	Topology	Permutations presentation	Partial duality
Graphs on surfaces, Ribbon graphs	1.1	1.2	1.3
Hypermaps	2.1	2.2	2.3
dessins d'Enfants	3.1	3.2	3.3 ???

orientable case

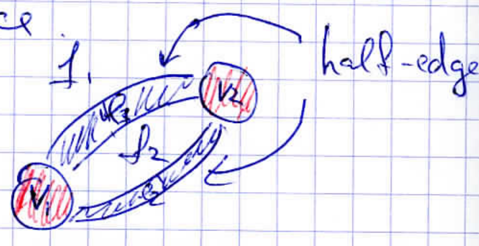
non orientable case

1.1

Ribbon graph (map) = cellular embedding of a graph into a (orientable) surface

the faces are cells = discs

vertices = discs , ribbon ~~=~~ discs

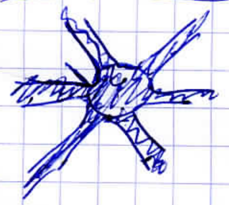


(Robert Cori 1975)

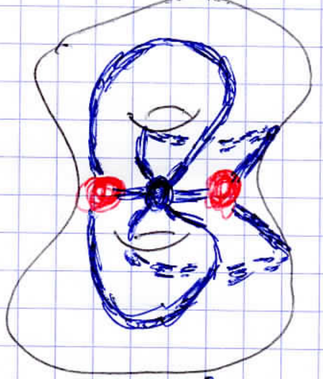
2.1

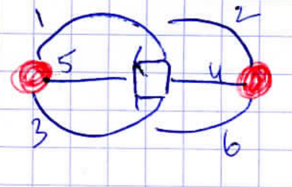
Hypermaps , a hyperedge has any number of half-edges.

a hyperedge



Example



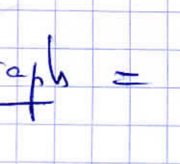


From 2.2 → *

1.2

Graph = involution σ_1, on the set of half-edges without fixed points

$$\sigma_E = (12)(34)(56)$$



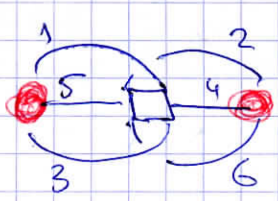
$\sigma_E = (12)(34)$
$\sigma_V = (13)(24)$

$\sigma_V = (1423)$
$\sigma_E = (12)(34)$

ribbon graph = $(\sigma_V, \sigma_E, \sigma_F)$ such that $\sigma_V \sigma_E \sigma_F = 1$

$$\sigma_V = (153)(246) \qquad \sigma_E^2 = 1$$

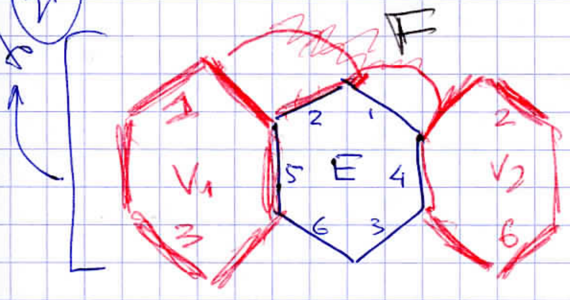
Hypermap $= (\sigma_V, \sigma_E, \sigma_F)$ such that $\sigma_V \sigma_E \sigma_F = 1$



$\sigma_V = (135)(246)$

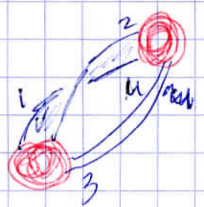
$\sigma_E = (125634)$

2,1



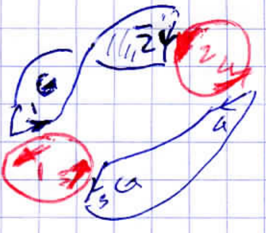
Hypermap = cell decomposition of the surface into cells of 3 colors: V, E, F

i,j,n



$\sigma_V = (13)(24)$

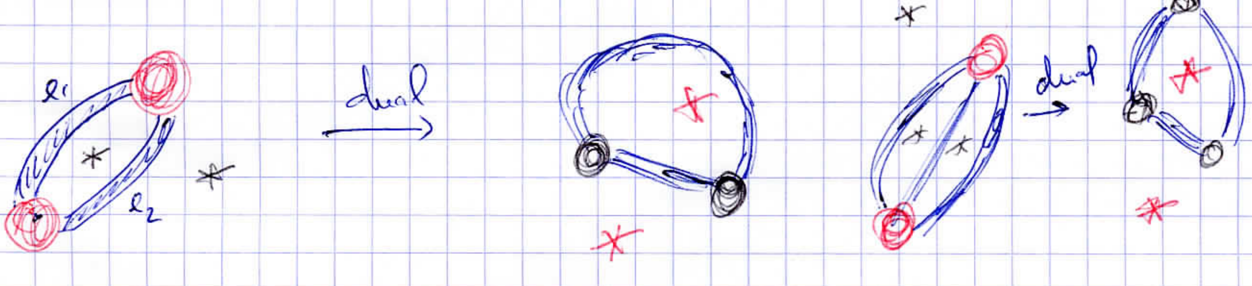
$\sigma_E = (1,\bar{2})(34) \in B_N = \mathrm{Sym}([-1,1]^N)$



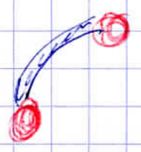
$\sigma_F = \cancel{\sigma_E^{-1} \sigma_V^{-1}} = (1423)$

$\sigma_V \sigma_E \sigma_F = (3\bar{4})$

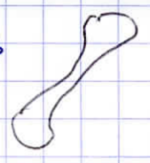
1.3



$\Gamma^{e_1} = \{$ 1. spanning subgraph Γ_{e_1} with all vertices and an edge e_1

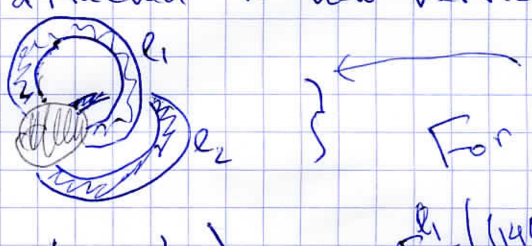


2. ~~boundary~~ boundary components



= vertices of Γ^e

3. edges Γ^e = edges of Γ (doubles) attached to new vertices



For this example

$\Gamma^{E'} = (\sigma_V \sigma_{E'}, \sigma_{E'}^{-1} \sigma_{\bar{E}} \ldots)$

$F^{e_1} = ((1423), (12)(34), \ldots)$

2.3 For hypermaps

$$\Gamma^{E'} = \left(\sigma_V \sigma_{E'},\ \sigma_{E'}^{-1} \sigma_{\overline{E'}},\ \sigma_{E'} \sigma_F\right)$$

$$\Gamma^{V'} = \left(\sigma_{V'}^{-1} \sigma_{\overline{V'}},\ \sigma_{V'} \sigma_E,\ \sigma_F \sigma_{V'}\right)$$

$$\Gamma^{F'} = \left(\sigma_{F''} \sigma_V,\ \sigma_E \sigma_{F'},\ \sigma_{F'}^{-1} \sigma_{\overline{F'}}\right)$$

No bar