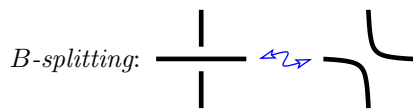
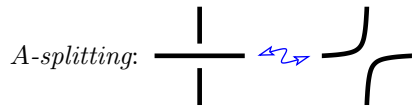


# Knots & Links

## The Kauffman bracket and the Jones polynomial [Ka1].

Let  $L$  be a link diagram.



A state  $S$  is a choice of either  $A$ - or  $B$ -splitting at every classical crossing.

$$\alpha(S) = \#(\text{of } A\text{-splittings in } S)$$

$$\beta(S) = \#(\text{of } B\text{-splittings in } S)$$

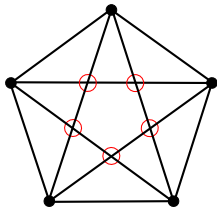
$$\delta(S) = \#(\text{of circles in } S)$$

$$[L](A, B, d) := \sum_S A^{\alpha(S)} B^{\beta(S)} d^{\delta(S)-1}$$

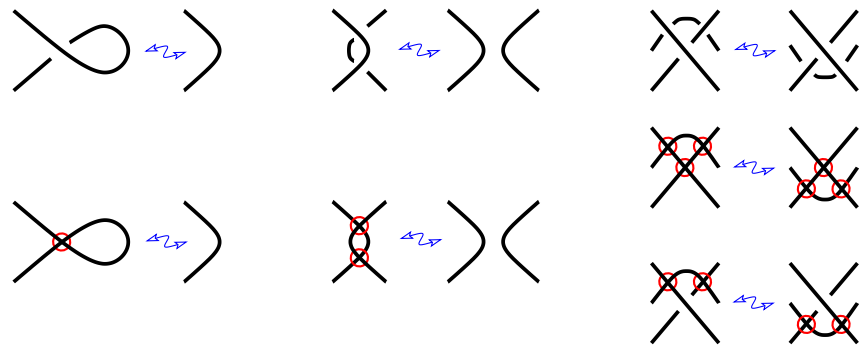
$$J_L(t) := (-1)^{w(L)} t^{3w(L)/4} [L](t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2})$$

## Virtual links [Ka3].

Virtual crossings



Reidemeister moves

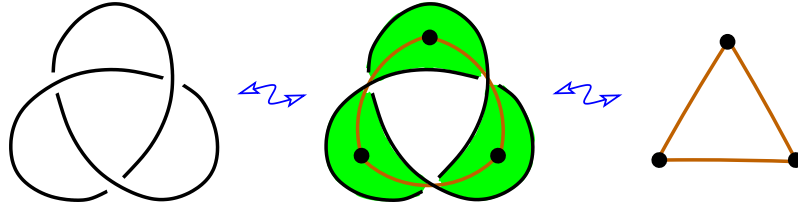


Example

$(\alpha, \beta, \delta)$	$(3, 0, 1)$	$(2, 1, 2)$	$(2, 1, 2)$	$(1, 2, 1)$
$(2, 1, 2)$	$(1, 2, 1)$	$(1, 2, 3)$	$(0, 3, 2)$	

$$[L] = A^3 + 3A^2Bd + 2AB^2 + AB^2d^2 + B^3d; \quad J_L(t) = 1$$

**Thistlethwaite's Theorem** [Ka1] *Up to a sign and multiplication by a power of  $t$  the Jones polynomial  $J_L(t)$  of an alternating link  $L$  is equal to the Tutte polynomial  $T_\Gamma(-t, -t^{-1})$ .*



The theorem was generalized to non-alternating links (using signed graphs in [Ka2] and using the Bollobás-Riordan polynomial for ribbon graphs in [DFKLS]); and virtual links in [ChVo].

**Theorem** [ChVo]. *Let  $L$  be an virtual link and  $G_L$  be the corresponding ribbon graph. Then*

$$[L](A, B, d) = A^n B^r d^{k-1} R_{G_L} \left( \frac{Ad}{B}, \frac{Bd}{A}, \frac{1}{d} \right) .$$

#### REFERENCES

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- [Ka2] L. H. Kauffman, *A Tutte polynomial for signed graphs*, Discrete Appl. Math. **25** (1989) 105–127.
- [Ka3] L. Kauffman, *Virtual knot theory*, European Journal of Combinatorics, **20** (1999) 663–690.