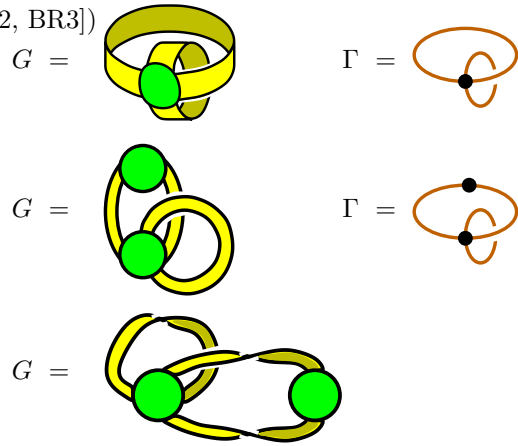


Ribbon graphs (B. Bollobás and O. Riordan [BR2, BR3])

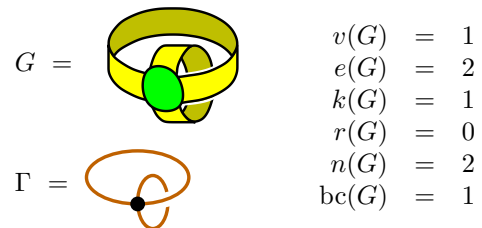
A ribbon graph G is a surface represented as union of vertices-discs and edges-ribbons such that

- discs and ribbons intersect by disjoint line segments,
- each such line segment lies on the boundary of precisely one vertex and precisely one edge;
- every edge contains exactly two such line segments.



The Bollobás-Riordan polynomial [BR2, BR3]

- $v(G)$ be the number of vertices of G ;
- $e(G)$ be the number of its edges of G ;
- $k(G)$ be the number of components of G ;
- $r(G) := v(G) - k(G)$ be the rank of G ;
- $n(G) := e(G) - r(G)$ be the nullity of G ;
- $bc(G)$ be the number of connected components of the boundary of G ;



$$R_G(x, y, z) := \sum_F x^{r(G)-r(F)} y^{n(F)} z^{k(F)-bc(F)+n(F)}$$

$$R_G(x, y, z) = 1 + 2y + y^2 z^2$$

Example

(k, r, n, bc)	$(2, 0, 0, 2)$	$(1, 1, 0, 1)$	$(1, 1, 0, 1)$	$(2, 0, 1, 2)$
	$(1, 1, 1, 2)$	$(1, 1, 1, 1)$	$(1, 1, 1, 1)$	$(1, 1, 2, 1)$

$$R_G(x, y, z) = x + 2 + xyz + y + 2yz + y^2 z^2 .$$

Relations to the Tutte polynomial

$R_G(x - 1, y - 1, 1) = T_\Gamma(x, y)$. If G is planar (genus zero): $R_G(x - 1, y - 1, z) = T_\Gamma(x, y)$.

REFERENCES

[BR2] B. Bollobás, O. Riordan, *A polynomial of graphs on orientable surfaces*, Proc. London Math. Soc. **83** (2001) 513–531.
 [BR3] B. Bollobás, O. Riordan, *A polynomial of graphs on surfaces*, Math. Ann. **323**(1) (2002) 81–96.