Math 6112 – Spring 2020
Problem Set 4
Due: Friday 7 February 2020

15. Show that the contravariant Hom functor $Hom_R(-, N)$ from $R-\underline{mod}$ to $\mathbb{Z}-\underline{mod}$ is left exact.

16. Let $M = \mathbb{Z}$ and $N = \mathbb{Z}/m\mathbb{Z}$ with $m > 1$ and let $\nu \in Hom_{\mathbb{Z}}(M, N)$ be the canonical homomorphism from M to N. Show that id_N cannot be written as $\nu \circ f$ for any $f \in Hom_{\mathbb{Z}}(N, M)$. Hence show that the image of the exact sequence $M \to N \to 0$ under $Hom_{\mathbb{Z}}(N, -)$ is not exact.

17. Let $M = \mathbb{Z}$ and $N = m\mathbb{Z}$ with $m > 1$ and let $\iota \in Hom_{\mathbb{Z}}(N, M)$ be the injection of N into M. Show that the homomorphism $\rho : N \to M$ given by $\rho(mx) = x$ cannot be written as $g \circ \iota$ for any $g \in Hom_{\mathbb{Z}}(M, M)$. Hence show that the image of the exact sequence $0 \to N \to M$ under $Hom_{\mathbb{Z}}(-, M)$ is not exact.

18. Show that if m is a positive integer then $(\mathbb{Z}/m\mathbb{Z}) \otimes_{\mathbb{Z}} (m\mathbb{Z}) \simeq \mathbb{Z}/m\mathbb{Z}$. Hence show that

$$0 \longrightarrow (m\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z}/m\mathbb{Z}) \xrightarrow{i \otimes 1} \mathbb{Z} \otimes_{\mathbb{Z}} (\mathbb{Z}/m\mathbb{Z})$$

is not exact, where i is the canonical injection $m\mathbb{Z} \hookrightarrow \mathbb{Z}$. Use this to conclude that that $M \otimes -$ and $- \otimes N$ need not be exact functors.

19. Let I be an index set and $\{N_\alpha \mid \alpha \in I\}$ be a set of left R-modules. Let $N = \bigoplus_\alpha N_\alpha$ be their direct sum. Show that N is flat iff each N_α is flat.