Math 6112 – Spring 2020
Problem Set 6
Due: 21 February 2020

25. (Baer) Prove that an R-module Q is injective iff any homomorphism of a left ideal $\mathfrak{a}$ of R into Q can be extended to a homomorphism of R into Q.

26. If D is an integral domain, then a D-module T is *divisible* iff for all $a \in D$, $a \neq 0$, the map $a_T : T \to T$ given by $a_T(t) = at$ is surjective. Show that any injective module over an integral domain is divisible. If in addition D is a PID show that any divisible module is injective. (In particular, an abelian group T is divisible iff it is injective as a $\mathbb{Z}$-module.)

27. Given the Snake Diagram as in class,

$$\begin{array}{ccccccccc} & & M' & \xrightarrow{f} & M & \xrightarrow{g} & M'' & \longrightarrow & 0 \\ & & {\scriptstyle d'}\downarrow & & {\scriptstyle d}\downarrow & & {\scriptstyle d''}\downarrow & & \\ 0 & \longrightarrow & N' & \xrightarrow{h} & N & \xrightarrow{k} & N'' & & \end{array}$$

(a) Show that if d' and d'' are injective, then so is d.

(b) Show that if d' and d'' are surjective, then so is d.

(c) Show that if the rows are actually short exact sequences, so that in addition f is injective and k is surjective, then if any two of d', d, and d'' are isomorphisms, then so is the third.