Some useful formulas

Some trigonometric formulas

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad , \quad \cos^2 x = \frac{1+\cos 2x}{2} \quad , \quad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Some integration formulas

$$\int \sqrt{x^2+1}\, dx = \frac{1}{2}\, x\sqrt{1+x^2} + \frac{1}{2}\, \ln\left(x + \sqrt{1+x^2}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \ln\left(x + \sqrt{1+x^2}\right) + C$$

Conics

Ellipse/Hyperbola $x^2/a^2 \pm y^2/b^2 = 1$

In the case $a > b$*:* denote $c = \sqrt{a^2 - b^2}$ for ellipse and $c = \sqrt{a^2 + b^2}$ for hyperbola and, for $a > b$: Eccentricty: $e = c/a$ Foci: $(\pm c, 0)$

In the case $a < b$*:* the roles of x and y are exchanged in the formulas above.

Parabola: $x^2 = 4py$ Focus $(0, p)$.

Polar coordinates

Element of length: $ds = \sqrt{dr^2 + r^2 d\theta^2}$

Area bounded by $r = r(\theta)$, $\theta = \theta_0, \theta = \theta_1$ is $A = \displaystyle\int_{\theta_0}^{\theta_1} \frac{1}{2} r^2 d\theta$

Angle ψ: $\tan\psi = \dfrac{r}{dr/d\theta}$

Rotation of the rectangular coordinate axes Oxy by angle θ to new coordinate axes $Ox'y'$: $x = x'\cos\theta - y'\sin\theta, \quad y = x'\sin\theta + y'\cos\theta$