

Note on convergent series

You don't need the limit comparison test to prove the following:

Suppose $\sum a_n$ and $\sum b_n$ are nonnegative convergent series. Then $\sum a_n b_n$ converges.

Proof. Since $\sum b_n$ converges, $\lim_{n \rightarrow \infty} b_n = 0$; so, eventually $b_n < 1$. Therefore, eventually, $a_n b_n < a_n \cdot 1 = a_n$ and $\sum a_n b_n < \sum a_n$. So, $\sum a_n b_n$ converges by the usual Comparison Test. \square