Metric Classification of Geometries of Positive Ricci Curvature in 3D

Vic Pătrângenaru
Department of Mathematics and Statistics
Texas Tech University,Lubbock, TX 79409-1042, U.S.A.
vpatrang@math.ttu.edu

March 29, 2005

2000 Mathematics Subject Classification: 53C30, 53C21, 58D17

Using Cartan's approach, [C1946], we give an explicit formula of the metric tensor of a simply connected 3D homogeneous space of positive scalar curvature in terms of the principal Ricci curvatures at one point. The principal Ricci curvatures at a single point fully determine such an isometry class of a 3D Riemannian homogeneous space, and each isometry class can be represented by Lie group with a left invariant metric. The problem raised in [P2002] that a 3D compact connected simply connected manifold has a Lie group structure is an open alternative to the Ricci flow approach [Pe2002] [Pe2003] to Poincaré's conjecture.

References

[C1946] Cartan, É. *Leçons sur la Géométrie des Espaces de Riemann.* (French) 2d ed. Gauthier-Villars, Paris, (1946).

[P2004] Pătrangenaru, V., Correction to: "On the 3D Riemannian homogeneous spaces of positive sectional curvature" *Algebra Geom. Appl. Semin. Proc. 3/4 (2004)*, 19-21.

[P2002] Pătrngenaru, V., On the 3D Riemannian homogeneous spaces of positive sectional curvature. Algebra Geom. Appl. Semin. Proc. 2 (2002), 5-13.

[Pe2002] Perelman, G., The entropy formula for the Ricci flow and its geometric applications. (2002) $http://arxiv.org/abs/math.DG/0211159$

[Pe2003] Perelman, G., Ricci flow with surgery on three-manifolds. (2003) $http://arxiv.org/abs/math.DG/0303109$