[Sample Final Exam, 132 Au08]

MATH 132
FINAL
WINTER 05
FORM C

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Soc. Sec.#: xxx-xx-xxxx
Rec. Instructor: Dong
Rec. Time: 1:30 - 3:18

Do all work in the space provided and place answers in the indicated spaces. Answers with no (or improper) support will not be given credit.

1. Evaluate the following Integrals

[$\int_0^1 \frac{e^{\sqrt{3x+1}}}{\sqrt{3x+1}}\,dx$, Sp08]

(a)

$$\int \frac{(\ln(x))^5}{2x}\,dx$$

(15)

Let $u = \ln x$ $\quad du = \frac{1}{x}\,dx$

$$\int \frac{(\ln x)^5}{2x}\,dx = \int \frac{u^5}{2}\,du$$

$$= \frac{1}{2}\cdot\frac{u^6}{6} + C$$

$$= \frac{1}{12}(\ln x)^6 + C$$

Answer 1(a): $\frac{1}{12}(\ln x)^6 + C$

1 (b) $\int \frac{7x^8 - 5x^6 + 3}{x}\,dx$ $\left[\int \frac{5\sqrt{x}}{\sqrt{7x^{\frac{6}{5}}+1}}\,dx,\ \text{Sp}08 \right]$

(15)

$$\int 7x^7 - 5x^5 + 3\,\frac{1}{x}\,dx$$

$$= \frac{7}{8}x^8 - \frac{5}{6}x^6 + 3\ln x + C$$

Answer: 1(b) $\frac{7}{8}x^8 - \frac{5}{6}x^6 + 3\ln x + C$

1 (c) $\int_0^1 (5x^4 + 2x)\sqrt{4x^5 + 4x^2 + 3}dx$ $\left[\int \frac{3 + 4e^x + 7e^{4x}}{e^x}\,dx \right]$

(15)

Let $u = 4x^5 + 4x^2 + 3$, then

$$du = (20x^4 + 8x)dx = 4(5x^4 + 2x)\,dx$$

$$dx = \frac{du}{4(5x^4+2x)}$$

2

$$\int (5x^4 + 2x)\sqrt{4x^5 + 4x^2 + 3}\,dx$$

$$= \int \cancel{(5x^4+2x)}\sqrt{u}\,\frac{du}{4\cancel{(5x^4+2x)}}$$

$$= \frac{1}{4}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{4}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{u^{\frac{3}{2}}}{6} + C = \frac{(4x^5+4x^2+3)^{\frac{3}{2}}}{6} + C$$

$$\int_0^1 (5x^4+2x)\sqrt{4x^5+4x^2+3}\,dx = \frac{(4x^5+4x^2+3)^{\frac{3}{2}}}{6}\Bigg|_0^1$$

$$= \frac{(4+4+3)^{\frac{3}{2}}}{6} - \frac{3^{\frac{3}{2}}}{6}$$

Answer: 1(c) $\frac{11^{\frac{3}{2}}}{6} - \frac{3^{\frac{3}{2}}}{6}$

2. The demand equation for a certain product is $p = \frac{143}{q+1}$, where p is the price per unit in dollars for q units. If the supply equation is $p = q + 3$, find

$$\left[\begin{cases} p = \frac{156}{q+3} \\ p = q+2 \end{cases}\right]$$

   (a) The equilibrium point
   (b) The Consumer's Surplus.

(16)

(a)

$$\frac{143}{q+1} = q+3 \qquad (q+1)(q+3) = 143$$

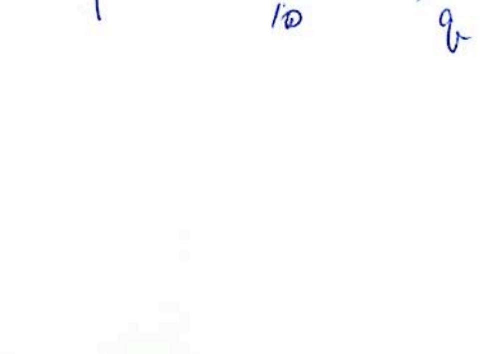
$$q^2 + 4q + 3 = 143 \qquad q^2 + 4q - 140 = 0$$

$$(q-10)(q+14) = 0$$

$$q = 10,\ -14$$

$$q_0 = 10, \quad p_0 = q_0 + 3 = 13$$

(b)



$$CS = \int_0^{10} \left(\frac{143}{q+1} - 13\right) dq$$

$$= \left[143\ln(q+1) - 13q\right]\Big|_0^{10}$$

$$= (143\ln(11) - 13\cdot 10) - (143\ln 1 - 13\cdot 0)$$

$$= 143\ln 11 - 130$$

Answer 2: (a) Equilibrium point : $(10, 13)$

(b) Consumer's Surplus: $143 \ln 11 - 130$

3. Use the provided tables on the last page of the exam to find the following integral. Please state the formula number you use.

$\left[\int \frac{5}{3x^2+7x^3}\, dx \right]$

(a)

$$\int \frac{2}{\sqrt{25x^2+9}}dx \quad \text{Let } u = 5x \quad a = 3 \quad dx = \frac{1}{5}du$$

(12)

$$= 2\int \frac{1}{\sqrt{u^2+a^2}} \frac{1}{5}du = \frac{2}{5}\int \frac{1}{\sqrt{u^2+a^2}}du$$

$$= \frac{2}{5}\ln\left|u + \sqrt{u^2+a^2}\right| + C$$

$$= \frac{2}{5}\ln\left|5x + \sqrt{25x^2+9}\right| + C$$

Answer 3(a): Formula number: 10

Answer:

$$\left[\int \frac{7}{x^5\sqrt{3-x^8}}\,dx \right]$$

(b)

(12) $\int x^3\sqrt{2+x^2}dx$

Let $u=x^2$ $du=2x\,dx$ $dx=\frac{1}{2x}du$

$\int x^3\sqrt{2+u}\ \frac{1}{2x}du = \frac{1}{2}\int u\sqrt{2+u}\,du$

$(a=2 \quad b=1)$ Formlar 6

$= \frac{1}{2}\ \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C$

$= \frac{(3u-4)(2+u)^{3/2}}{15} + C$

$= \frac{(3x^2-4)(2+x^2)^{\frac{3}{2}}}{15} + C$

Answer 3(b): Formula number: 6

Answer:__________________________

4. (a) Given $y' = e^{7x-2}$ and $y(1) = 10$, find y.

(15) $y = \int e^{7x-2}dx = \frac{e^{7x-2}}{7} + C$

$\frac{e^{7\cdot1-2}}{7} + C = 10$

$C = 10 - \frac{e^5}{7}$

$y = \frac{e^{7x-2}}{7} + 10 - \frac{e^5}{7}$

$$\left[\frac{1}{x^2}y' - 3x^5y = 0,\quad y>0,\quad y(1)=1 \right]$$ Sp 08

4(b)

Answer 4(a);

$$\left[\begin{array}{l} f(x,y) = (x^2+y^4)e^{5x^2y} \\ \text{Find } f_{xy}(x,y) \end{array}\right]$$

(b) Let $f(x) = \sqrt[3]{x}$, use differentials to estimate $\sqrt[3]{127}$.

(10)

$$f(x+dx) \approx f(x) + f'(x)\,dx$$

$$f(x) = \sqrt[3]{x} \qquad x = 125 \qquad dx = 2$$

$$\sqrt[3]{127} \approx \sqrt[3]{125} + \frac{1}{3}\,125^{-\frac{2}{3}} \cdot 2$$

$$= 5 + \frac{1}{3} \cdot \frac{1}{25} \cdot 2$$

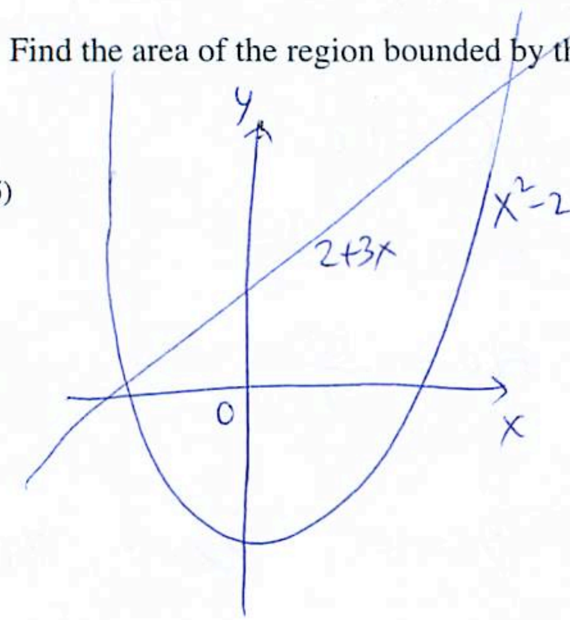
$$= 5 + \frac{2}{75}$$

$$= \frac{377}{75}$$

Answer 4(b): ____________________

5. Find the area of the region bounded by the graphs of $f(x) = x^2 - 2$ and $g(x) = 2 + 3x$.

(15)



$$\left[\begin{array}{l} y^2 = x \\ x+3y = 4 \end{array}\right]$$

$$x^2-2=2+3x$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x=4,\ -1$$

$$\int_{-1}^{4}\left[(2+3x)-(x^2-2)\right]dx$$

$$=\int_{-1}^{4}(-x^2+3x+4)\,dx$$

$$=\left(-\frac{x^3}{3}+3\frac{x^2}{2}+4x\right)\Big|_{-1}^{4}$$

$$=\left(-\frac{64}{3}+3\cdot 8+16\right)-\left(\frac{1}{3}+\frac{3}{2}-4\right)$$

$$=40-\frac{64}{3}-\frac{1}{3}-\frac{3}{2}+4$$

$$=\frac{125}{6}$$

Answer 5: ____________________

7 6. Determine if the following integral is convergent or divergent. If it is convergent,

then

find its value.

(15) $$\int_0^{\infty} xe^{2x^2-1}dx$$

Let $u = 2x^2 - 1$ $\quad du = 4x\,dx$

$dx = \frac{1}{4x}\,du$

$$\int xe^{2x^2-1}\,dx = \int x e^u \frac{1}{4x}\,du$$

$$= \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C$$

$$= \frac{1}{4}e^{2x^2-1} + C$$

$$\int_0^{\infty} xe^{2x^2-1}\,dx = \lim_{r\to\infty}\int_0^r xe^{2x^2-1}\,dx$$

$$= \lim_{r\to\infty}\left(\frac{1}{4}e^{2x^2-1}\right)\Big|_0^r$$

$$= \lim_{r\to\infty}\left(\frac{1}{4}e^{2r^2-1} - \frac{1}{4}e^{0-1}\right)$$

$$= \infty$$

divergent Answer 6: Convergent or divergent:

Value : _______________

7 (a). Suppose $f(x,y) = x^5 \ln(e^x + 3y) + y^9(x + 8)$.

Find $f_{xy}(x,y)$.

(10)

$$f_x(x,y) = 5x^4 \ln(e^x+3y) + x^5 \frac{1}{e^x+3y} \cdot e^x + y^9$$

$$f_{xy}(x,y) = 5x^4 \frac{1}{e^x+3y} \cdot 3 + x^5 e^x \left(-\frac{1}{(e^x+3y)^2}\right) \cdot 3 + 9y^8$$

$$= \frac{15x^4}{e^x+3y} - \frac{3x^5e^x}{(e^x+3y)^2} + 9y^8$$

Answer 7(a): _______________________

[$f(x,y,z) = 3x^2 - 2y^2 + z^2$ $\quad 6x + 4y - 3z = 26$]

(b) Use the method of Lagrange multipliers to determine the critical points of $f(x,y,z) = x^2 + 2y^2 + z^2$ subject to the constraint $2x + y + 3z = 10$.

(15)

$$F(x,y,z,\lambda) = f(x,y,z) - \lambda\, g(x,y,z)$$

$$= x^2 + 2y^2 + z^2 - \lambda\,(2x + y + 3z - 10)$$

$F_x = 2x - 2\lambda = 0$ $\qquad x = \lambda$

$F_y = 4y - \lambda = 0$ $\qquad y = \frac{1}{4}\lambda$

$F_z = 2z - 3\lambda = 0$ $\qquad z = \frac{3}{2}\lambda$

plug in

$F_\lambda = -(2x + y + 3z - 10) = 0$

$$2\lambda + \frac{1}{4}\lambda + 3\frac{3}{2}\lambda = 10$$

$$\frac{27}{4}\lambda = 10$$

$$\lambda = \frac{40}{27}$$

$$\left(\frac{40}{27}, \frac{10}{27}, \frac{60}{27}\right)$$

Answer 7(b): ____________________

8. Suppose z is a function of x and y. Use implicit differentiation to find $\frac{\partial z}{\partial x}$

from the equation $2x^3y + 5x^2z^4 + 3y^2z - 9x + 11 = 0$. $[\, e^{x^2z} + 3yz^3 - 9x + 1 = 0 \,]$

(15)

$$6x^2y + 10xz^4 + 20x^2z^3\frac{\partial z}{\partial x} + 3y^2\frac{\partial z}{\partial x} - 9 = 0$$

$$(20x^2z^3 + 3y^2)\frac{\partial z}{\partial x} = -6x^2y - 10xz^4 + 9$$

$$\frac{\partial z}{\partial x} = \frac{-6x^2y - 10xz^4 + 9}{20x^2z^3 + 3y^2}$$

9. Find all the critical points of the following function. By using the second derivative test,

determine whether the critical point corresponds to a relative maxima, a relative minima,

a saddle point or whether the test gives no information

$$f(x,y) = 4x^2 - 8xy + \frac{1}{3}y^3 + 15y + 2 \qquad \left[f(x,y) = y^3 + x^2 - 9y^2 - 6x - 24y + 1 \right]$$

{Second derivative test: If (x_0,y_0) is a critical point and $D(x,y) = f_{xx}(x,y)\, f_{yy}(x,y) - [\, f_{xy}]^2$

Then a) $D(x_0,y_0) > 0$ and $f_{xx}(x_0,y_0) > 0$ then (x_0,y_0) is a point of rel minima

b) $D(x_0,y_0) > 0$ and $f_{xx}(x_0,y_0) < 0$ then (x_0,y_0) is a point of rel max

c) $D(x_0,y_0) < 0$ then (x_0,y_0) is a saddle point

d) $D(x_0,y_0) = 0$ then no conclusion can be drawn.

$f_x(x,y) = 8x - 8y = 0$ ①

$f_y(x,y) = -8x + y^2 + 15 = 0$ ②

From ① $x = y$

(20) Plug in ② $y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$

$y = 3, 5$

$x = 3, 5$

Since $x = y$, only two critical points

$(3,3)$ $(5,5)$

$f_{xx}(x,y) = 8$ $f_{yy}(x,y) = 2y$

$f_{xy}(x,y) = -8$

$D(x,y) = 8 \cdot 2y - (-8)^2 = 16y - 64$

$D(3,3) = 16 \cdot 3 - 64 = -16 < 0$

12 $(3,3)$ is a saddle point

$D(5,5) = 16 \cdot 5 - 64 = 16 > 0$

$f_{xx}(5,5) = 8 > 0$

$(5,5)$ is a rel. min point.

1. $\int \frac{du}{a + bu} = \frac{1}{b} \ln |a + bu| + C$

2. $\int \frac{u\,du}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln |a + bu| + C$

3. $\int \frac{u^2\,du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln |a + bu| + C$

4. $\int \frac{du}{u(a+bu)} = \frac{1}{a} \ln \left|\frac{u}{a+bu}\right| + C$

5. $\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln\left|\frac{a+bu}{u}\right| + C$

6. $\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C$

7. $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C$

8. $\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C$

9. $\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2 \ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

10. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

11. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln\left|\frac{a+u}{a-u}\right| + C$

12. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln\left|\frac{u-a}{u+a}\right| + C$

13. $\int e^u\,du = e^u + C$

14. $\int a^u\,du = \frac{a^u}{\ln a} + C : a > 0, a \neq 1$

15. $\int \sqrt{\frac{a+u}{b+u}}\,du = \sqrt{(a+u)(b+u)} + (a-b)\ln(\sqrt{a+u} + \sqrt{b+u}) + C$

16. $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

17. $\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$