MATH 132
MIDTERM III
WINTER 06
FORM A

Name: DONG DU
Soc. Sec.#: XXX-XX-XXXX
Rec. Instructor: Dong Du
Rec. Time: 12:30-1:18

Do all work in the space provided and circle the answers. Answers with no (or improper) support will not be given credit.

I. Determine if the following integral is convergent or divergent. If it is convergent then find its value

$$\int_e^\infty \frac{(\ln(x))^3}{x}\ dx$$

(15)

Let $u = \ln(x)$, then $du = \frac{1}{x}dx$

$$\int \frac{(\ln(x))^3}{x}dx = \int u^3\,du = \frac{u^4}{4} + C$$

$$= \frac{(\ln(x))^4}{4} + C$$

$$\int_e^\infty \frac{(\ln(x))^3}{x}dx = \lim_{r\to\infty}\int_e^r \frac{(\ln(x))^3}{x}dx$$

$$= \lim_{r\to\infty} \frac{(\ln(x))^4}{4}\Big|_e^r$$

$$= \lim_{r\to\infty}\left[\frac{(\ln(r))^4}{4} - \frac{(\ln(e))^4}{4}\right]$$

$$= \frac{\left(\lim_{r\to\infty}\ln(r)\right)^4}{4} - \frac{1}{4}$$

$$= \infty$$

Answer 1 : Convergent or Divergent : Divergent

Value : ______

2. For the function $z = \dfrac{\ln(3x^2 - xy + 5)}{2x^3y^4 - 7}$, find $\dfrac{\partial z}{\partial x}$.

(15)

$$\frac{\partial z}{\partial x} = \frac{\frac{\partial}{\partial x} \ln(3x^2 - xy + 5) \cdot (2x^3y^4 - 7) - \ln(3x^2 - xy + 5) \cdot \frac{\partial}{\partial x}(2x^3y^4 - 7)}{(2x^3y^4 - 7)^2}$$

$$= \frac{\frac{1}{3x^2 - xy + 5} \cdot (6x - y) \cdot (2x^3y^4 - 7) - \ln(3x^2 - xy + 5) \cdot 6x^2y^4}{(2x^3y^4 - 7)^2}$$

$$= \frac{6x - y}{(3x^2 - xy + 5)(2x^3y^4 - 7)} - \frac{6x^2y^4 \ln(3x^2 - xy + 5)}{(2x^3y^4 - 7)^2}$$

$$\frac{6x - y}{(3x^2 - xy + 5)(2x^3y^4 - 7)} - \frac{6x^2y^4 \ln(3x^2 - xy + 5)}{(2x^3y^4 - 7)^2}$$

Answer 2: ________________

3. The demand functions $q_A, q_B$ for the products A and B are given by

$$q_A = e^{-(3p_A+2p_B)} \text{ and } q_B = \frac{10}{(p_A)^3(p_B)^3},$$ where $q_A, q_B$ are the number of units of A and B demanded when the unit prices are $p_A$, $p_B$ respectively.

(a) Determine whether the products are competitive,complementary or neither.

(15 pts.)

$$\frac{\partial q_A}{\partial p_B} = e^{-(3p_A+2p_B)} \cdot \frac{\partial}{\partial p_B}\left(-(3p_A+2p_B)\right) = -2e^{-(3p_A+2p_B)} < 0$$

$$\frac{\partial q_B}{\partial p_A} = \frac{10}{p_B^3}\frac{\partial}{\partial p_A}(p_A^{-3}) = \frac{10}{p_B^3}(-3)p_A^{-4} = -\frac{30}{p_A^4 p_B^3} < 0$$

Answer: 3(a) complementary

(b) If the unit prices of A and B are \$1.00 and \$2.00 respectively, estimate the change in the demand for A when the price of B is decreased by \$0.15 and the price of A is held constant.

$$\Delta q_A \approx dq_A = \frac{\partial q_A}{\partial p_A} dp_A + \frac{\partial q_A}{\partial p_B} dp_B$$

The price of A is held constant, so $dp_A = 0$.

$dp_B = -0.15$

$$\left.\frac{\partial q_A}{\partial p_B}\right|_{(1,2)} = -2e^{-(3\cdot 1+2\cdot 2)} = -2e^{-7}$$

$$\Delta q_A \approx -2e^{-7}\cdot(-0.15) = 0.3e^{-7}$$



Answer 3(b): $0.3e^{-7}$

4. Suppose z is a function of x and y. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ from the equation

$$\ln(zx^3) + 9x^3z^2 - 2yz + 3xy + 12y = \frac{4}{x}$$

(20) $\frac{\partial}{\partial x}\left(\ln(zx^3) + 9x^3z^2 - 2yz + 3xy + 12y\right) = \frac{\partial}{\partial x}\left(\frac{4}{x}\right)$

$$\frac{1}{zx^3}\left(\frac{\partial z}{\partial x}\cdot x^3 + z\cdot 3x^2\right) + 27x^2z^2$$

$$+18x^3z\frac{\partial z}{\partial x} - 2y\frac{\partial z}{\partial x} + 3y = -\frac{4}{x^2}$$

$$\underline{\frac{1}{z}\cdot\frac{\partial z}{\partial x}} + \frac{3}{x} + 27x^2z^2 \underline{+18x^3z\frac{\partial z}{\partial x}}$$

$$\underline{-2y\frac{\partial z}{\partial x}} + 3y = -\frac{4}{x^2}.$$

$$\left(\frac{1}{z} + 18x^3z - 2y\right)\frac{\partial z}{\partial x} = -\frac{4}{x^2} - \frac{3}{x} - 27x^2z^2 - 3y$$

$$\frac{\partial z}{\partial x} = \frac{-\frac{4}{x^2} - \frac{3}{x} - 27x^2z^2 - 3y}{\frac{1}{z} + 18x^3z - 2y}$$

Answer 4: $\dfrac{-\frac{4}{x^2} - \frac{3}{x} - 27x^2z^2 - 3y}{\frac{1}{z} + 18x^3z - 2y}$

$$= (24x^7z^2 + 12x^8yz^3 + x^9y^2z^4)\, e^{xyz} + \frac{(3-3xy-3x^2y^2)\, e^{xy}}{(e^{xy}+3xyz)^2} - \frac{(6y-6xy^2)(xe^{xy}+3xz)\, e^{xy}}{(e^{xy}+3xyz)^3}$$

final answer!

5. Supoose $w = f(x,y,z)$ is a function of x,y,and z given by

$$w = f(x,y,z) = z^2x^7e^{xyz} + \ln(e^{xy} + 3xyz) + 2x^2y^3z^2 + 7y.$$

Find ~~$f_{xyz}(x,y,z)$~~. $f_{xzy}(x,y,z)$

(15)

$$f_x = \frac{\partial}{\partial x}(z^2x^7)\cdot e^{xyz} + z^2x^7\cdot\frac{\partial}{\partial x}(e^{xyz}) + \frac{1}{e^{xy}+3xyz}\cdot\frac{\partial}{\partial x}(e^{xy}+3xyz) + 4xy^3z^2$$

$$= 7x^6z^2e^{xyz} + z^2x^7\cdot e^{xyz}\cdot yz + \frac{1}{e^{xy}+3xyz}(e^{xy}\cdot y + 3yz) + 4xy^3z^2$$

$$= 7x^6z^2e^{xyz} + x^7yz^3e^{xyz} + \frac{ye^{xy}+3yz}{e^{xy}+3xyz} + 4xy^3z^2$$

$$f_{xz} = 14x^6z\, e^{xyz} + 7x^6z^2\cdot xy\, e^{xyz} + 3x^7yz^2e^{xyz} + x^7yz^3\cdot xy\cdot e^{xyz} + \frac{3y(e^{xy}+3xyz) - 3xy(ye^{xy}+3yz)}{(e^{xy}+3xyz)^2} + 8xy^3z$$

$$= 14x^6z\, e^{xyz} + \overbrace{7x^7yz^2e^{xyz} + 3x^7yz^2e^{xyz}}^{\text{Combine}} + x^8y^2z^3e^{xyz} + \frac{(3y-3xy^2)e^{xy} + \cancel{9xy^2z} - \cancel{9xy^2z}}{(e^{xy}+3xyz)^2}$$

$$= (14x^6z + 10x^7yz^2 + x^8y^2z^3)\, e^{xyz} + \frac{(3y-3xy^2)\, e^{xy}}{(e^{xy}+3xyz)^2}$$

$$f_{xzy} = (10x^7z^2 + 2x^8yz^3)\, e^{xyz} + (14x^6z + 10x^7yz^2 + x^8y^2z^3)\cdot e^{xyz}\cdot(xz) + \frac{\frac{\partial}{\partial y}\left((3y-3xy^2)\, e^{xy}\right)(e^{xy}+3xyz)^2}{(e^{xy}+3xyz)^4} - \frac{(3y-3xy^2)\, e^{xy}\frac{\partial}{\partial y}\left((e^{xy}+3xyz)^2\right)}{(e^{xy}+3xyz)^4}$$

Answer 5 :

look up

5

$$= (10x^7z^2 + 2x^8yz^3 + 14x^7z^2 + 10x^8yz^3 + x^9y^2z^4)\, e^{xyz} + \frac{[(3-6xy)\, e^{xy} + (3y-3xy^2)\, x e^{xy}]}{(e^{xy}+3xyz)^2} - \frac{(3y-3xy^2)\, e^{xy}\cdot \cancel{2(e^{xy}+3xyz)}\cdot(xe^{xy}+3xz)}{(e^{xy}+3xyz)^{\cancel{4}\,3}}$$

6 . Find all the critical points of the following function. By using the Second Derivative Test, determine whether the critical point corresponds to a relative maximum, a relative minimum or a saddle point or whether the test gives no information

$$f(x,y) = 3x^2 + 12xy + 2y^3 - 6x + 6y.$$

{Second Derivative Test: If $(x_0 y_0)$ is a critical point and

$$D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - \left[f_{xy}(x,y)\right]^2$$

Then a) if $D(x_0 y_0) > 0$ and $f_{xx}(x_0 y_0) > 0$ then $(x_0 y_0)$ is a point of rel minimum
b) if $D(x_0 y_0) > 0$ and $f_{xx}(x_0 y_0) < 0$ then $(x_0 y_0)$ is a point of rel maximum
c) if $D(x_0 y_0) < 0$ then $(x_0 y_0)$ is a saddle point
d) if $D(x_0 y_0) = 0$ then no conclusion can be drawn.

$f_x = 6x + 12y - 6$ $\quad\left\{\begin{array}{l} 6x+12y-6=0 \\ 12x+6y^2+6=0 \end{array}\right.$

$f_y = 12x + 6y^2 + 6$

(20)

$\Rightarrow \left\{\begin{array}{l} x+2y-1=0 \\ 2x+y^2+1=0 \end{array}\right.$ $\quad x = 1-2y$

$2(1-2y)+y^2+1=0$

$y^2-4y+3=0 \quad (y-1)(y-3)=0 \quad y=1$ or $3$

$x = 1-2\cdot 1 = -1$ or $x = 1-2\cdot 3 = -5$

Critical points $(-1,1), (-5,3)$.

$f_{xx} = 6 \quad f_{yy} = 12y \quad f_{xy} = 12$

$D(x,y) = f_{xx}(x,y)\cdot f_{yy}(x,y) - [f_{xy}(x,y)]^2$

$= 6\cdot 12y - 12^2 = 72y - 144 = 72(y-2)$

$D(-1,1) = 72(1-2) = -72 < 0$ $\quad (-1,1)$ saddle point

$D(-5,3) = 72(3-2) = 72 > 0$

$f_{yy}(-5,3) = 12\cdot 3 = 36 > 0$ $\quad (-5,3)$ rel min point

Answer 6: $(-1,1)$ saddle point
$(-5,3)$ rel min point

MATH 132
MIDTERM III
SPRING 07
FORM B

Name: DONG DU
Soc. Sec.#: xxx - xx - xxxx
Rec. Instructor: Dong Du
Rec. Time: 11:30-12:18

Do all work in the space provided and circle the answers. Answers with no (or improper) support will not be given credit.

I. Determine if the following integral is convergent or divergent. If it is convergent then find its value

$$\int_0^{\infty} \frac{x^2}{\left(4x^3+1\right)^4}\,dx$$

(12)

Let $u = 4x^3+1$, then $du = 12x^2 dx$

$$dx = \frac{du}{12x^2}$$

$$\int \frac{x^2}{(4x^3+1)^4}\,dx = \int \frac{x^2}{u^4}\frac{du}{12x^2}$$

$$= \frac{1}{12}\int u^{-4}\,du$$

$$= \frac{1}{12}\cdot\frac{u^{-3}}{-3} + C$$

$$= -\frac{1}{36}(4x^3+1)^{-3} + C$$

$$\int_0^{\infty}\frac{x^2}{(4x^3+1)^4}\,dx = \lim_{r\to\infty}\int_0^r \frac{x^2}{(4x^3+1)^4}\,dx$$

$$= \lim_{r\to\infty} -\frac{1}{36}(4x^3+1)^{-3}\Big|_0^r$$

$$= \lim_{r\to\infty}\left[-\frac{1}{36}(4r^3+1)^{-3} + \frac{1}{36}(4\cdot 0^3+1)^{-3}\right]$$

$$= \frac{1}{36}$$

Answer I : Convergent or Divergent : Convergent

Value : $\frac{1}{36}$

2(a). For the function $z = \frac{5x^3 + y}{3x^2y^5 + 1}$, find $\frac{\partial z}{\partial x}$.

(6 pts)

$$\frac{\partial z}{\partial x} = \frac{\frac{\partial}{\partial x}(5x^3+y)\cdot(3x^2y^5+1) - (5x^3+y)\frac{\partial}{\partial x}(3x^2y^5+1)}{(3x^2y^5+1)^2}$$

$$= \frac{15x^2\cdot(3x^2y^5+1) - (5x^3+y)\cdot 6xy^5}{(3x^2y^5+1)^2}$$

$$= \frac{45x^4y^5 + 15x^2 - 30x^4y^5 - 6xy^6}{(3x^2y^5+1)^2}$$

Ans. 2(a) $\frac{15x^4y^5 + 15x^2 - 6xy^6}{(3x^2y^5+1)^2}$

2(b). For the function $z = x^3\ln(7x^5 + y)$, find $\frac{\partial z}{\partial x}$.

(6 pts)

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^3)\cdot\ln(7x^5+y) + x^3\cdot\frac{\partial}{\partial x}\ln(7x^5+y)$$

$$= 3x^2\ln(7x^5+y) + x^3\cdot\frac{1}{7x^5+y}\cdot 35x^4$$

$$= 3x^2\ln(7x^5+y) + 35\frac{x^7}{7x^5+y}$$

Ans.2(b) $3x^2\ln(7x^5+y) + \frac{35x^7}{7x^5+y}$

2(c). For the function $z = e^{7x^4y^2+5x+3}$, find $\frac{\partial z}{\partial x}$.

(6 pts)

$$\frac{\partial z}{\partial x} = e^{7x^4y^2+5x+3}\cdot\frac{\partial}{\partial x}(7x^4y^2+5x+3)$$

$$= (28x^3y^2+5)\,e^{7x^4y^2+5x+3}$$



Ans. 2(c): $(28x^3y^2+5)\,e^{7x^4y^2+5x+3}$

3. The demand functions $q_A$, $q_B$ for the products A and B are given by

$$q_A = \frac{40}{(p_A)^3}(p_B)^4 \text{ and } q_B = \frac{110}{(p_B)^5}\sqrt[3]{p_A},$$ where $q_A, q_B$ are the number of

units of A and B demanded when the unit prices are $p_A$, $p_B$ respectively.

(a) Determine whether the products are competitive,complementary or neither

(8 pts.)

$$\frac{\partial q_A}{\partial P_B} = \frac{40}{P_A^3}\cdot 4\, P_B^3 = 160 \cdot \frac{P_B^3}{P_A^3} > 0$$

$$(\ast) \quad \frac{\partial q_B}{\partial P_A} = \frac{110}{(P_B)^5}\cdot \frac{1}{3} P_A^{-\frac{2}{3}} = \frac{110}{3\, P_B^5 \cdot P_A^{\frac{2}{3}}} > 0$$

Answer: 3(a) competitive

(b) If the unit prices of A and B are \$27.00 and \$2.00 respectively, estimate the change in the demand for B when the price of A is decreased from \$27.00 to \$25.00 and the price of B is held constant.

(7 pts)

$$\Delta q_B \approx dq_B = \frac{\partial q_B}{\partial P_A} dP_A + \frac{\partial q_B}{\partial P_B} dP_B$$

Since the price of B is held constant, $dP_B = 0$.

$$(\ast) \quad \left.\frac{\partial P_B}{\partial P_A}\right|_{(27,2)} = \frac{110}{3 \cdot 2^5 \cdot 27^{\frac{2}{3}}} = \frac{110}{3 \cdot 32 \cdot 9} = \frac{110}{864}$$

$$dP_A = 25 - 27 = -2$$

$$\Delta q_B \approx \frac{110}{864}\cdot(-2) = -\frac{220}{864} = -\frac{55}{216}$$

Answer 3(b); $-\frac{55}{216}$

4. (a) Suppose z is a function of x and y. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ from the equation

$$4x^5y^3 + 3x^5z - 7x + y^2 - 9 = 0$$

(10 pts.)

$$\frac{\partial}{\partial x}\left(4x^5y^3 + 3x^5z - 7x + y^2 - 9\right) = 0$$

$$20x^4y^3 + 15x^4z + 3x^5\frac{\partial z}{\partial x} - 7 = 0$$

$$3x^5\frac{\partial z}{\partial x} = -20x^4y^3 - 15x^4z + 7$$

$$\frac{\partial z}{\partial x} = \frac{-20x^4y^3 - 15x^4z + 7}{3x^5}$$

Answer 4(a) $\frac{-20x^4y^3 - 15x^4z + 7}{3x^5}$

4(b). Suppose z is a function of x and y. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ from the equation: $e^{x^3y+z} + 2z^6 - 7x^2y + 1 = 0$.

(10 pts)

$$\frac{\partial}{\partial x}\left(e^{x^3y+z} + 2z^6 - 7x^2y + 1\right) = 0$$

$$e^{x^3y+z}\cdot\left(3x^2y + \frac{\partial z}{\partial x}\right) + 12z^5\frac{\partial z}{\partial x} - 14xy = 0$$

$$3x^2y\,e^{x^3y+z} + e^{x^3y+z}\frac{\partial z}{\partial x} + 12z^5\frac{\partial z}{\partial x} - 14xy = 0$$

$$\left(e^{x^3y+z} + 12z^5\right)\frac{\partial z}{\partial x} = -3x^2y\,e^{x^3y+z} + 14xy$$

$$\frac{\partial z}{\partial x} = \frac{-3x^2y\,e^{x^3y+z} + 14xy}{e^{x^3y+z} + 12z^5}$$

Ans.4(b) $\frac{-3x^2y\,e^{x^3y+z} + 14xy}{e^{x^3y+z} + 12z^5}$

5. Suppose $w = f(x,y,z)$ is a function of x,y,and z given by

$$w = f(x,y,z) = x\ln(5y+2z) + 3x^4yz + e^{4x+5y+2z}.$$

Find $f_{xyz}(x,y,z)$.

(15)

$$f_x = \ln(5y+2z) + 12x^3yz + 4e^{4x+5y+2z}$$

$$f_{xy} = \frac{1}{5y+2z}\cdot 5 + 12x^3z + 4\cdot 5e^{4x+5y+2z}$$

$$f_{xyz} = -\frac{5}{(5y+2z)^2}\cdot 2 + 12x^3 + 4\cdot 5\cdot 2e^{4x+5y+2z}$$

$$= -\frac{10}{(5y+2z)^2} + 12x^3 + 40e^{4x+5y+2z}$$

Answer 5 : $-\frac{10}{(5y+2z)^2} + 12x^3 + 40e^{4x+5y+2z}$

6 . Find all the critical points of the following function. By using the Second Derivative Test, determine whether the critical point corresponds to a relative maximum, a relative minimum or a saddle point or whether the test gives no information

$$f(x,y) = x^3 - 3x^2 - 9x - y^2 + 6.$$

{Second Derivative Test: If $(x_0,y_0)$ is a critical point and

$$D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - \left[f_{xy}(x,y)\right]^2$$

Then a) if $D(x_0,y_0) > 0$ and $f_{xx}(x_0,y_0) > 0$ then $(x_0,y_0)$ is a point of rel minimum
b) if $D(x_0,y_0) > 0$ and $f_{xx}(x_0,y_0) < 0$ then $(x_0,y_0)$ is a point of rel maximum
c) if $D(x_0,y_0) < 0$ then $(x_0,y_0)$ is a saddle point
d) if $D(x_0,y_0) = 0$ then no conclusion can be drawn.

(20)

$$f_x = 3x^2 - 6x - 9$$

$$f_y = -2y$$

$$\begin{cases} 3x^2 - 6x - 9 = 0 \\ -2y = 0 \end{cases} \Rightarrow \begin{cases} 3(x^2 - 2x - 3) = 0 \\ y = 0 \end{cases}$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x_1 = 3, \quad x_2 = -1$$

So critical points $(3,0)$, $(-1,0)$

$f_{xx} = 6x - 6$, $f_{yy} = -2$, $f_{xy} = 0$

$D(x,y) = (6x-6)(-2) - 0^2 = -12x + 12$

$D(3,0) = -12 \cdot 3 + 12 < 0$, $(3,0)$ saddle point

$D(-1,0) = -12 \cdot (-1) + 12 = 24 > 0$, $f_{xx}(-1,0) = 6 \cdot (-1) - 6 = -12 < 0$

$(-1,0)$ rel max

Answer 6 : $(3,0)$ saddle point

$(-1,0)$ rel max point