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# RELATIVE TRACE FORMULA AND SIMPLE ALGEBRAS

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ABSTRACT. A Deligne-Kazhdan variant of the relative trace formula of Jacquet-Lai is given, thus generalizing the study of distinguished representations from the context of the quaternion algebra to that of any simple algebra.

**1.** Let $F$ be a global field, $E$ a separable quadratic extension, $\mathbf{A} = \mathbf{A}_F$ and $\mathbf{A}_E$ the associated rings of adeles, and $G$ a reductive $F$-group. Let $L(G)$ be the space of square-integrable functions $\varphi$ on $G(E)\backslash G(\mathbf{A}_E)$ such that for any proper parabolic subgroup $P$ of $G$ with unipotent radical $N$ we have $\int \varphi(nx)\,dn = 0$ ($n$ in $N(E)\backslash N(\mathbf{A}_E)$) for any $x$ in $G(\mathbf{A}_E)$. An irreducible constituent of the representation $r$ of $G(\mathbf{A}_E)$ on $L(G)$ by right translations is called a *cuspidal* $G(\mathbf{A}_E)$-module. A cuspidal $G(\mathbf{A}_E)$-module $\pi$ is called *distinguished* if there is an integrable function $\varphi$ in the space of $\pi$ such that the integral $B(\varphi) = \int \varphi(x)\,dx$, on the closed subset $G(F)\backslash G(\mathbf{A})$ of $G(E)\backslash G(\mathbf{A}_E)$, is nonzero.

Let $M$ be a simple algebra of dimension $n^2$ central over $F$, where $n \geqslant 2$. Then there is a division algebra $D$ of rank $d$ dividing $n$ central over $F$ so that $M$ is the matrix algebra $M(m, D)$ of $m$ by $m$ matrices over $D$, where $n = dm$. Let $G$ be the quotient of the multiplicative group of $M$ by its center. Let $S$ be the set of places $v$ of $F$ where $D$ is ramified. Assume that each $v$ in $S$ splits in $E$. Let $G'$ be the quotient of $\mathrm{GL}(n)$ by its center. For each place $v$ of $F$ we write $F_v$ for the completion of $F$ at $v$, and $E_v = E \otimes_F F_v$. For each $v$ outside $S$ we have $G_v \simeq G'_v$, where $G_v = G(F_v)$, $G'_v = G'(F_v)$. Fix two places $u$ and $u'$ of $F$ with $u'$ in $S$. Denote by $u$ and $u'$ also a fixed place of $E$ above $u$ and $u'$. Let $\pi$ be a cuspidal $G(\mathbf{A}_E)$-module which corresponds (by the Deligne-Kazhdan correspondence (see [**F**])) to a cuspidal $G'(\mathbf{A}_E)$-module $\pi'$, whose component $\pi'_u$ at $u$ is supercuspidal, and at $u'$ it is discrete-series. Thus $\pi_v \simeq \pi'_v$ via $\overline{G}_v \simeq \overline{G}'_v$ for any place $v$ outside $S$. We put $\overline{G}_v = G(E_v)$, $\overline{G}'_v = G'_v(E_v)$.

THEOREM. *$\pi$ is distinguished if and only if $\pi'$ is distinguished.*

The case of $n = 2$, where $\pi'_u$ is assumed to be only discrete-series, is due to Jacquet and Lai [**JL**]. For general $n$ the condition at $u$ can be removed on applying furthur computations of the trace formula; but this will not be discussed here. The integrals $B(\varphi)$ were first studied by Shimura and Asai [**A**]. Applications of distinguished representations to the theory of Euler products are given in [**F′**].

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**2.** For every place $v$ of $F$ put $G_v = G(F_v)$, and let $R_v$ be the ring of integer in $E_v$. For $v$ outside $S$ put $K'_v = G(R_v)$ and $K_v = K'_v \cap G_v$, and let $f_v^0$ be the quotient by the volume $|K_v|$ of the characteristic function of $K'_v$. Fix a differential form of highest weight defined over $F$, hence a product measure $dx = \otimes dx_v$ on $G(\mathbf{A})$, so that the product of the volumes $|K_v|$ converges. Let $f = \otimes f_v$ be a function on $G(\mathbf{A}_E)$ such that for every place $v$ the component $f_v$ is a complex valued compactly supported function on $\overline{G}_v = G(E_v)$, which is locally constant if $v$ is finite, or smooth if $v$ is archimedean, with $f_v = f_v^0$ for almost all $v$. Since $G'$ is an inner form of $G$ we obtain a matching measure on $G'(\mathbf{A})$, also denoted by $dx$. We choose a function $f' = \otimes f'_v$ on $G'(\mathbf{A}_E)$, with the above properties, so that (1) at any $v$ outside $S$ we take $f'_v = f_v$ via $\overline{G}'_v \simeq \overline{G}_v$, (2) at any $v$ in $S$ which splits into $v'$, $v''$ in $E$, the convolutions $h'_v = f'^*_{v'} * f'_{v''}$ and $h_v = f^*_{v'} * f_{v''}$ have matching orbital integrals. Here $f^*_{v'}(g) = f_{v'}(g^{-1})$,

$$(f^*_{v'} * f_{v''})(g) = \int f^*_{v'}(gx^{-1}) f_{v''}(x)\,dx,$$

and our requirement is that for any regular $x$ in $G_v \simeq G_{v'} \simeq G_{v''}$, and $x'$ in $G'_v$, which have the same sets of eigenvalues, we have

$$\int h'_v(g^{-1}x'g)\,dg = \int h_v(g^{-1}xg)\,dg.$$

The integrals are over $Z'_v(x) \backslash G'_v$ and $Z_v(x) \backslash G_v$, where $Z_v(x)$ is the centralizer of $x$ in $G_v$, and the isomorphic tori $Z_v(x)$ and $Z'_v(x)$ are given matching measures.

Further, at some finite place $u$ of $E$ we require $f'_u$, and $f_u$, to be supercuspidal. Namely, for any $E_u$-parabolic subgroup of $G'_u$ with unipotent radical $N_u$, and any $x$, $y$ in $G'_u$, the integral $\int f'_u(xny)\,dn$ is 0. Hence the operator $r'(f') = \int f'(x) r'(x)\,dx$ ($x$ in $G'(E) \backslash G'(\mathbf{A}_E)$) on $L(G')$ is an integral operator with kernel $K'(x, y) = \sum_\gamma f'(x^{-1}\gamma y)$ ($\gamma$ in $G'(E)$).

An element $\gamma$ of $G'(E)$ is called regular if it has distinct eigenvalues, and elliptic if it lies in a compact torus of $G'(\mathbf{A}_E)$. Thus $\gamma$ is elliptic regular if it lies in no proper $E$-parabolic subgroup of $G'(E)$. We say that $\gamma$ in $G'(E)$ is *relatively regular* (resp. *elliptic*) if $\sigma(\gamma)\gamma^{-1}$ is regular (resp. elliptic) in $G'(E)$. We denote by $\sigma$ the nontrivial element of $\mathrm{Gal}(E/F)$. Note that the centralizer of a regular $\sigma(\gamma)\gamma^{-1}$ is defined over $F$. From now on we deal only with relatively regular elements $\gamma$ in $G'(E)$.

LEMMA. *Let $A$ be either a torus or a parabolic subgroup of $G'$ defined over $F$. Then $\gamma$ lies in $A(E)G'(F)$ if and only if $\sigma(\gamma)\gamma^{-1}$ lies in $A(E)$.*

PROOF. Suppose $A$ is parabolic over $F$. Then the first cohomology group $H^1(\mathrm{Gal}(E/F), A(E))$ is trivial. If $p = \sigma(\gamma)\gamma^{-1}$ lies in $A(E)$, we have $p\sigma(p) = 1$. The cocycle $a_\sigma = p$ then splits, namely there is $u$ in $A(E)$ with $p = \sigma(u)u^{-1}$. Hence $g = \sigma(u^{-1}\gamma) = u^{-1}\gamma$ lies in $G'(F)$, and $\gamma = ug$ is in $A(E)G'(F)$.

Suppose $p$ is elliptic regular. Let $\theta$ be an element of $E - F$ with $\theta^2$ in $F$. Then $E = F(\theta) = F(\theta + c)$ for all $c$ in $F$. Put $\gamma = \gamma_1 + \theta\gamma_2$, for $\gamma_i$ in $G'(F)$. Consider the polynomial $P(c) = \det(\gamma_1 + c\gamma_2)$. If $P(c) = 0$ for all $c$ in $F$ then $P$ is identically zero, and $P(\theta) = 0$, contrary to the assumption that $\det\gamma \neq 0$. Hence there is $c$ in $F$ so that $\gamma_1' = \gamma_1 + c\gamma_2$ is invertible. Put $\theta' = \theta - c$. Then $\gamma = \gamma_1' + \theta'\gamma_2$. Define $\delta = \gamma\gamma_1'^{-1}$, and $\delta' = \gamma_2\gamma_1'^{-1}$. Then $\delta = 1 + \theta'\delta'$. If $\delta'$ is nonelliptic in $G'(F)$, then it lies in a parabolic subgroup of $G'(F)$, hence $\delta$ lies in a parabolic of $G'(E)$ defined over $F$, and so does $p = \sigma(\delta)\delta^{-1}$, contrary to our assumption. Hence $\delta'$ lies in an elliptic torus $T(F)$ of $G'(F)$, $\delta$ lies in $T(E)$, and so does $p$. But $\delta$ commutes with the regular $p$, whose centralizer in $G'(E)$ is $T(E)$. The Lemma follows.

We say that $\gamma$ and $\gamma'$ in $G'(E)$ are *relatively conjugate* if there are $x, y$ in $G'(F)$ with $\gamma' = x\gamma y$; equivalently (by the Lemma), if $\sigma(\gamma)\gamma^{-1}$ and $\sigma(\gamma')\gamma'^{-1}$ are conjugate by an element of $G'(F)$. Indeed, by the Lemma we can assume that both $\gamma$ and $\gamma'$ lies in $A(E)$.

Let $\{T\}$ be a set of representatives for the conjugacy classes of elliptic tori in $G'(F)$. Put $\bigcup_T T(E)/\sim$ for the quotient of the relatively regular subset of $\bigcup_T T(E)$ by the equivalence relation: $t$ in $T(E)$ is equivalent to $t'$ in $T'(E)$ if $T = T'$, and there is $w$ in the normalizer of $T(F)$ in $G(F)$, and $t''$ in $T(F)$, with $t^w = t't''$.

COROLLARY. (1) $\gamma$ *in* $G'(E)$ *is relatively elliptic if and only if it lies in* $G'(F)T(E)G'(F)$, *where T is an elliptic torus of* $G'$ *defined over F.*

(2) $\bigcup_T T(E)/\sim$ *is a set of representatives for the relative conjugacy classes of the relatively elliptic regular elements in* $G'(E)$.

Assume that $f'$ is such that $f'(x\gamma y)$ is 0 for any $x, y$ in $G'(\mathbf{A})$ and $\gamma$ in $G'(E)$, unless $\gamma$ is relatively elliptic regular. Assume that $f$ satisfies the analogous condition. For example, we can make the analogous local assumption on the components $f_u''$ and $f_{u'}$ at a place $u'$ of $S$.

3. PROPOSITION. *We have* $\int\int K(x, y)\,dx\,dy = \int\int K'(x', y')\,dx\,dy$; $x, y$ *range over* $G(F)\backslash G(\mathbf{A})$; $x', y'$ *are over* $G'(F)\backslash G'(\mathbf{A})$.

PROOF. The map which associates to $g$ in $\mathrm{GL}(n, \mathbf{A}_E)$ the coefficients $\{a_i;\ a_n = \det g\}$ in the characteristic polynomial of $g$ yields an isomorphism from the variety of semisimple conjugacy classes in $G(\mathbf{A}_E)$ to the quotient of $\mathbf{A}_E^{n-1} \times \mathbf{A}_E^\times$ by $\mathbf{A}_E^\times$, where $\{a_i\} \simeq \{a_i z^i\}$ ($z$ in $\mathbf{A}_E^\times$). Suppose $f(x^{-1}\gamma x) \neq 0$ ($\gamma$ in $G(E)$; $x, y$ in $G(\mathbf{A})$). Then the image of $x^{-1}\sigma(\gamma)\gamma^{-1}x$ lies in a compact subset of $\mathbf{A}_E^{n-1} \times \mathbf{A}_E^\times/\mathbf{A}_E^\times$, and also in the discrete subset $E^{n-1} \times E^\times/E^\times$, hence in a finite set. Consequently, only finitely many relative conjugacy classes (of relatively elliptic regular) $\gamma$ contribute to the sum $\sum f(x^{-1}\gamma x)$ over $\gamma$ in $G(E)$, which defines $K(x, y)$.

Replacing $f$ by its absolute value, we see that the integrals below are absolutely convergent; hence the rearrangements below are justified. Then

$$\begin{aligned} K(x, y) &= \sum_T \ \sideset{}{'}\sum_{\gamma \in T(E)} \ \sum_{\alpha \in G(F)/T(F)} \ \sum_{\beta \in N(T)\backslash G(F)} f(x^{-1}\alpha\gamma\beta y) \\ &= \sum_T w(T)^{-1} \sideset{}{'}\sum_{\gamma \in T(E)/T(F)} \ \sum_{\alpha \in G(F)} \ \sum_{\beta \in T(F)\backslash G(F)} f(x^{-1}\alpha\gamma\beta y), \end{aligned}$$

where $N(T)$ is the normalizer of $T(F)$ in $G(F)$, $w(T)$ is the cardinalilty of its Weyl group, and the prime over $\Sigma$ indicates summation over relatively regular elements only. Integrating over $x$, $y$ in $G(F)\backslash G(\mathbf{A})$, we obtain

$$\iint K(x,y)\,dx\,dy = \sum_T \frac{|T(\mathbf{A})/T(F)|}{w(T)} \sum_{\gamma\in T(E)/T(F)}{}' \iint f(x\gamma y)\,dx\,dy.$$

On the right $x$ ranges over $G(\mathbf{A})$, and $y$ over $T(\mathbf{A})\backslash G(\mathbf{A})$. $|T(\mathbf{A})/T(F)|$ is the volume of the compact group $T(\mathbf{A})/T(F)$.

Each of the integrals is a product of local integrals. If $v$ is a place of $F$ which does not split in $E$ and $G_v = G(F_v)$, $T_v = T(F_v)$, we obtain

$$\iint f_v(x\gamma y)\,dx\,dy \qquad (x \text{ in } G_v,\ y \text{ in } T_v\backslash G_v).$$

This converges. Indeed, if the integrand is nonzero, then $x\gamma y$ lies in a compact, hence $y^{-1}\sigma(\gamma^{-1})\gamma y$ is in a compact, hence $y$ is in a compact modulo $T(E_v)$ (since $\sigma(\gamma^{-1})\gamma$ is regular), hence $y$ is in a compact modulo $T_v$. But for such $y$ the function $x\mapsto f_v(x\gamma y)$ is compactly supported, and our integral converges.

At almost all nonsplit $v$ we have that $f_v = f_v^0$ is the quotient by $|K_v|$ of the characteristic function of $K_v'$, $E_v/F_v$ is unramified and $\gamma$ is in $K_v'$. If $f_v(x\gamma y)\neq 0$ then $x\gamma y$ is in $K_v'$, and so is $y^{-1}\sigma(\gamma^{-1})\gamma y$. But $\sigma(\gamma^{-1})\gamma$ is regular in $K_v'$. Hence $y$ lies in $T(E_v)K_v'\cap G_v$; since $E_v/F_v$ is unramified, the intersection is $T_vK_v$. Hence we can take $y$ in $K_v$ and conclude that $x$ is in $K_v$. Hence the integral is equal to the volume $|K_v|/|K_v\cap T_v|$ for almost all $v$ which do not split $E/F$.

If $v$ is a place of $F$ which splits into $v'$ and $v''$ in $E$, then $\gamma=(\gamma',\gamma'')$ in $G(E_v) = G_v\times G_v$, and our local integral is

$$\iint f_{v'}(x\gamma' y) f_{v''}(x\gamma'' y)\,dx\,dy \qquad (x \text{ in } G_v,\ y \text{ in } T_v\backslash G_v).$$

That is

$$\iint f_{v'}(x) f_{v''}(xy^{-1}\gamma'^{-1}\gamma'' y)\,dx\,dy = \int h_v(y^{-1}\delta y)\,dy,$$

where $h_v = f_{v'}^* * f_{v''}$, and $\delta = \gamma'^{-1}\gamma'' = \gamma^{-1}\sigma(\gamma)$ (we embed $G_v$ diagonally in $G(E_v)$). This is the orbital integral of $h_v$ at (the regular element) $\delta$, hence it converges.

For almost all such $v$, $f_{v'}$ and $f_{v''}$ are the characteristic functions of $K_v$ divided by $|K_v|$, and $\gamma',\gamma''$ lie in $K_v$. If the integrand is nonzero, then $x\gamma' y$, $x\gamma'' y$ are in $K_v$. Hence $y^{-1}\delta y$ is in $K_v$, and $y$ is in $T_vK_v$. Taking $y$ in $K_v$, it follows that $x$ is in $K_v$, and the integral is equal to $|K_v|/|T_v\cap K_v|$ once again.

These computations hold for any reductive group $G$, in particular for our $G$ and $G'$, and the equality of the proposition follows since we have the same sums, volume factors, and integrals on both sides. Indeed, at $v$ outside $S$ we take $f_v = f_v'$, and at $v$ in $S$ we take $f_v$ and $f_v'$ so that $h_v$ and $h_v'$ have matching (regular) orbital integrals, and this is precisely what is needed for the comparison of the local integrals.

**4. Proof of theorem.** Suppose $\pi$ is distinguished. We claim that so is $\pi'$. The opposite direction is similar. Now if $v$ is a place of $F$ which splits into $v'$ and $v''$ in $E$, then the restriction of $B$ to the space of $\pi$ is a nonzero $G(\mathbf{A})$-invariant form, which can be restricted to a nonzero $G_{v'} \times G_{v''}$-invariant linear form on the space of $\pi_{v'} \times \pi_{v''}$; hence $\pi_{v''}$ is the contragredient of $\pi_{v'}$, and we write $\pi_v$ for $\pi_{v'}$ so that $\pi_{v''} = \check{\pi}_v$.

Let $S'$ be a sufficiently large set of places of $F$ containing $S$ and the archimedean places, so that the space of $\pi$ contains a nonzero vector invariant under the action of $K^{S'} = \prod_v K'_v$ ($v$ outside $S'$). For $v$ outside $S'$ we take a $K'_v$-bi-invariant $f_v$. Then $\pi^{S'}(f^{S'})$ factors through a projection from the space $V$ of $\pi$ to the space $V(\pi)$ of $K^{S'}$-fixed vectors in $V$, and it acts on $V(\pi)$ as a scalar. The space $V(\pi)$ is $G_{S'}$-invariant, where $G_{S'} = \prod_v G_v$ ($v$ in $S'$), and the associated representation of $G_{S'}$ is denoted by $\pi_{S'}$. Hence for $\varphi$ in $V(\pi)$ we have

$$\pi(f)\varphi = \pi^{S'}(f^{S'}) \cdot \pi_{S'}(f_{S'})\varphi.$$

These comments apply to any cuspidal $G(\mathbf{A}_E)$-module $\rho$ with a supercuspidal component at $u$, so that we can write

$$K(x, y) = \sum_{\rho} \rho^{S'}(f^{S'}) \cdot K_\rho(x, y),$$

where $K_\rho$ is the kernel of $\rho_{S'}(f_{S'})$, and the sum is over all cuspidal $G(\mathbf{A}_E)$-modules which contain a nonzero $K^{S'}$-invariant vector, whose component at $u$ is supercuspidal. We have

$$\iint K(x, y)\,dx\,dy = \sum_{\rho} \rho^{S'}(f^{S'})a_\rho, \qquad a_\rho = \iint K_\rho(x, y)\,dx\,dy.$$

Note that $\pi_{S'} = \otimes \pi_v$ ($v$ in $S'$) is $\pi_S \otimes \pi_{S''}$, and $V(\pi) = V(S) \otimes V(S'')$, where $S''$ is the complement of $S$ in $S'$, and $\pi_S = \otimes \pi_v$, $V(S) = \otimes V_v$ ($v$ in $S$); a similar definition holds for $S''$.

We may assume that there is a unitary vector $t$ in the space $V(S'')$, such that the restriction of $B$ to $t \otimes V(S'')$ is nonzero. Choose $f_{S''}$ so that $\pi_{S''}(f_{S''})$ is the orthogonal projection on $t$. If $\{v_k\}$ is a basis of $V(S)$, then

$$\iint K_\pi(x, y)\,dx\,dy = \sum_k B(t \otimes \pi_S(f_S)v_k)\overline{B}(t \otimes v_k).$$

Indeed,

$$K_\pi(x, y) = \sum_k \pi_S(f_S)v_k(x)\bar{v}_k(y).$$

Note that if $u$ lies in $S''$ then we may assume that the component of $f_{S''}$ at $u$ is supercuspidal, since (1) if $g_u$ is supercuspidal then $f_u * g_u * f_u$ is supercuspidal, (2) if $\pi_u(f_u)$ is a projection on the space spanned by the unitary vector $t$, and $g_u(x) = (t, \pi_u(x)t)$, then $\pi_u(f_u * g_u * f_u) = \pi_u(f_u)\pi_u(g_u)\pi_u(f_u)$ is also a projection on the space spanned by $t$, multiplied by a nonzero scalar.

The space $V(S)$ is the tensor product of the spaces $V_{v'} \otimes V_{v''}$ over the $v$ in $S$, where $v', v''$ are the places of $E$ above $v$. The restriction of $B$ to $t \otimes V(S)$ is the tensor product of nonzero $G_v \times G_v$-invariant linear forms $C_v$ on $V_{v'} \otimes V_{v''}$, up to a constant. For each $v$ in $S$ choose bases $\{a_i\}$ and $\{b_j\}$ of $V_{v'}$ and $V_{v''}$ which are dual to each other with respect to $C_v$. The sum over $k$ is equal (up to a constant) to

$$\sum_{ij} C_v\left[\pi_{v'}(f_{v'})a_i \otimes \pi_{v''}(f_{v''})b_j\right]\overline{C}_v\left[a_i \otimes b_j\right].$$

As $C_v(a_i \otimes b_j) = \delta_{ij}$ we obtain

$$\sum_i C_v[\pi_v(f_{v'})a_i \otimes \check{\pi}_v(f_{v''})b_i] = \sum_i C_v[\pi_v(f_{v''})\pi_v(f_{v'})a_i \otimes b_i]$$
$$= \sum_i C_v[\pi_v(f_{v''} * f_{v'})a_i \otimes b_i] = \operatorname{tr}\pi_v(f_{v''} * f_{v'}).$$

If we replace $f_{v''}$ by $f_{v''} + f_{v''}^*$ we may assume that $f_{v''}^* = f_{v''}$ and that $h_v = f_{v''} * f_{v'}$. It is clear that there is $h_v$ which is zero on the singular set, in fact supported on the elliptic regular set if $\pi_v$ is discrete-series, so that $\operatorname{tr}\pi_v(h_v) \neq 0$. We choose such a function at $u'$.

Similar analysis applies in the case of $G'$. In particular,

$$\iint K'(x, y)\,dx\,dy = \sum_{\rho'} \rho'^{S'}(f'^{S'})a_{\rho'}, \qquad a_{\rho'} = \iint K'_{\rho'}(x, y)\,dx\,dy,$$

is equal to $\iint K(x, y)\,dx\,dy$ by the Proposition, for a function $f'$ related to $f$ as there. Applying linear independence of characters of the Hecke algebra of $G^{S'} \simeq G'^{S'}$, we conclude that $\pi^{S'}(f^{S'}) = \pi'^{S'}(f'^{S'})$ for our corresponding $\pi$, $\pi'$, and that $a_{\pi'} = a_\pi$ is nonzero. But $K'_{\pi'}$ is a sum of terms of the form $\varphi(x)\bar{\varphi}'(y)$, where $\varphi$, $\varphi'$ lie in the space of $\pi'$. Hence the restriction of $B'$ to the space of $\pi'$ is nonzero, and $\pi'$ is distinguished, as required.

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