[1] (4 points each) For the following problems, just write down the answers. No work is necessary. Right or wrong.

(a) If A is an 8×7 matrix and $\dim N(A) = 5$, then $\text{rank}(A) =$__________.

(b) If the eigenvalues of a 3×3 matrix A are -2, 1, and 3, then the characteristic polynomial of A is __________.

(c) If a matrix A is nonsingular, then A cannot have an eigenvalue ________.

(d) If the rank of a 4×5 matrix A is 3, then the rank of A^T is __________.

(e) If A is a 4×5 matrix and nullity of A^T is 1, then the nullity of A is __________.

(f) The dimension of the row space of a 8×10 matrix is at most __________.

(g) If U is an orthogonal matrix and x is a vector then the length of the vector Ux is__________.

(h) If $\{[3, 2, 2]^T, [-2, a, 0]^T, [b, c, -13]^T\}$ is an orthogonal basis of R^3, then $a =$__________, $b =$__________, $c =$__________.

(i) If S is an orthogonal st of 5 nonzero vectors in R^7, then the dimension of $\text{Span}(S)$ is __________.

(j) The number of solutions of the homogeneous 3×4 system of linear equations is either __________ or __________.

[2] (4 points each, -2 points for wrong answer) True/False: Just write T(rue) or F(alse).

(a) Any system of linear equations has at least one solution.

(b) If a square matrix A has no zero rows, then A has an inverse.

(c) If the product AB of two $n \times n$ matrices A and B has an inverse, then an inverse of BA also exists.

(d) If A^2 has an eigenvalue 1, then A must have an eigenvalue 1.

(e) Any nonzero square matrix has at least one nonzero eigenvalue.

(f) The zero vector can never be an eigenvector of any square matrix.

(g) If the characteristic polynomial of a 2×2 matrix A is $(X-1)(X-2)$, then A^{-1} exists.

(h) Any system of linear equations with more equations than the number of unknowns is always inconsistent.

(i) If an echelon form E of a matrix A does not have (columns of) free variables, then E must have a zero row.

(j) Any $n \times n$ matrix A has inifnitely many $n \times n$ matrices B such that $AB = BA$.

[3] Let $V = \text{Span}\{[3\ 1\ 1\ \ -1]^T, [2\ 5\ 1\ 0]^T, [1\ 2\ \ -2\ \ -1]^T\}$.

(a) (10) Use the Gram-Scmidt method to find an orthogonal basis of V.

(b) (10) Determine whether or not the vector $[3\ 7\ \ -1\ \ -1]^T$ is in V. If it is in V, then express it as a linear combination of the othogonal basis obtained in (a).

[4] Let

$$A = \begin{vmatrix} 1 & 5 & 1 & 1 & 0 \\ 1 & 5 & 2 & 4 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 2 & 10 & 0 & -4 & 0 \end{vmatrix}$$

(a) (5) Reduce A to its reduced echelon form.

(b) (5) Find a basis of the range space (= column space) of A.

(c) (5) Find a basis of the null space of A.

[5] Let

$$A = \begin{vmatrix} 2 & 1 & 4 \\ -1 & -2 & -1 \\ -1 & -1 & -3 \end{vmatrix}$$

(a) (10) Find the characteristic polynomial of A.

(b) (5) Find all eigenvalues of A.

(c) (10) Find all linearly independent eigenvectors of A belonging to each eigenvalue of A.

[6] (15) If A is a 6×6-matrix with four eigenvalues, and the eigenspaces of two of these eigenvalues are two dimensional each, must A be diagonalizable? Credit only for correct answer with correct explanation.

[7] Let

$$A = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

(a) (10) Compute $\det A$ using cofactor expansion. Is A invertible?

(b) (10) What are $\det(A^{-1})$ and $\det(A^2)$? $\det(3A)$ and $\det(A^T)$? Let B be the matrix obtained from A by adding 5 times its 3^{rd} row to 1^{st} row. Find $\det B$.

(c) (10) Find the inverse of A, if it exists.

[8] (15) Let $A = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$. Find all subspaces W of $\mathbb{R}^2$ such that if x lies in W, so does Ax.