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Abstract. Matching of (twisted) orbital integrals of corresponding spherical functions on a reductive p-adic group G and its (twisted) endoscopic group H is a prerequisite to lifting representations from H to G by means of a comparison of trace formulae. Kottwitz-Shelstad [KS] conjectured the precise form that the matching takes. This matching statement is called "the fundamental lemma" by Langlands, who proved it for $GL(2)$. It can be reduced to the case of the unit element of the Hecke algebra, denoted by 1_{K_G} and 1_{K_H}. The case of the unit elements also implies the transfer of orbital integrals for general locally constant compactly supported functions; see Waldspurger [W2] (in the non twisted case).

Kazhdan [K] initiated the study of the fundamental lemma in the higher rank case, proving the matching for $G = GL(n)$, H an elliptic torus, on introducing a decomposition of a compact element as a product of commuting absolutely semi simple and topologically unipotent elements. In [F7] a twisted analogue of Kazhdan decomposition is introduced, and used to prove the matching for the symmetric square lifting from $SL(2)$ to $PGL(3)$. The present paper establishes the matching for the unit elements 1_{K_G} and 1_{K_H} on $G = GL(4)$ and its endoscopic group $H = GSp(2)$, with respect to the twisting by "transpose-inverse" (as in [F7], with $GL(4)$ instead of $GL(3)$), in the stable case. A crucial ingredient is again the twisted Kazhdan decomposition, as is the definition of the norm in [KS], which we use.

Another key idea we borrow from Weissauer [We], who proved the matching between 1_K on $GSp(2)$ and 1_K on its endoscopic group $GL(2) \times GL(2)/Z$, on noticing that any elliptic torus in $GSp(2)$ lies in an intermediate subgroup $GL(2)'(= SO(4))$ of $GSp(2)$, and reducing the computation to one in $GL(2)'$ by means of a double coset decomposition $GL(2)' \backslash GSp(2)/K$. Thus by means of the twisted Kazhdan decomposition we reduce our twisted orbital integrals (in the main case, where the element is topologically unipotent) to the group of fixed points of the twisting, namely $Sp(2)$. Then we compute the integrals by means of the double coset decomposition $GL(2)' \backslash Sp(2)/K$, and apply a similar analysis to the non twisted integrals on $GSp(2)$.

Our work is entirely explicit. We exhibit a set of representatives for the twisted conjugacy classes in G, in families of types which we call (I), (II), (III), and (IV). We list those in the same stable twisted conjugacy class. The listing is done on computing the Galois hypercohomology groups used in [KS], or simply on using low brow Galois cohomology, but it is important for us to exhibit explicit representatives, not just to describe the abstract structure of the conjugacy classes within the stable class. Further we describe the norm map explicitly for each type, and find representatives for the stable conjugacy classes and the conjugacy classes in it, for $GSp(2)$. The stable orbital integral is simply the sum over the orbits in the stable orbit. Thus our computations can be used to compute the unstable orbital integrals. In the case of $GSp(2)$ we recover the results of Weissauer [We]. In the twisted case, this is done here too for all unstable twisted endoscopic groups. We compute all unstable orbital integrals of 1_K on the group $Sp(2)$, which has more endoscopic groups than $GSp(2)$, and deduce all endoscopic transfers of orbital integrals.

Key words: (twisted) orbital integrals, (twisted) endoscopic groups, trace formula, symplectic group, (twisted) stable conjugacy, Galois cohomology, absolutely semi simple, topologically unipotent compact elements, double coset decomposition.

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