# Vocab for Differential Equations

- A differential equation is **ordinary** if the unknown function is a real-valued function of a single independent variable. **Partial** differential equations may include functions of many variables.

- The **order** of a differential equation is the highest derivative that appears in the equation.

  **Example.** For instance, $f''(x) = -f(x)$ is second-order, while $f^{(17)}(x) = e^x$ is seventeenth order.

- An ordinary differential equation for the function $f(x)$ is **linear** if it is of the form

$$a_n(x)\, f^{(n)}(x) + \cdots + a_1(x)\, f'(x) + a_0(x)\, f(x) = g(x) \qquad (*)$$

  for functions $a_i(x)$ and a function $g(x)$.

- An ordinary differential equation for $f(x)$ is **homogeneous** if $f(x)$ (or a derivative thereof) appears in each term. A differential equation is **inhomogeneous** if there is a term without $f$ or its derivatives.

  **Example.** For instance, a linear ordinary differential equation is homogeneous precisely when $g(x) = 0$ in equation $(*)$.

- An **integrating factor** is something you multiply a differential equation by in order to make it solvable.

  **Example.** For instance, to solve a first-order inhomogeneous linear ordinary differential equation, namely,

$$f'(x) + a_0(x)\, f(x) = g(x)$$

  you should multiply by $e^{\int a_0(x)\, dx}$ so that

$$e^{\int a_0(x)\, dx}\, f'(x) + e^{\int a_0(x)\, dx}\, a_0(x)\, f(x) = e^{\int a_0(x)\, dx}\, g(x)$$

  and then

$$\frac{d}{dx}\left(e^{\int a_0(x)\, dx}\, f(x)\right) = e^{\int a_0(x)\, dx}\, g(x).$$

  Now you can antidifferentiate the right-hand side to find an equation for $f(x)$.