The Second Midterm

- There will be **12 questions** on the exam and it will last **50 minutes**.
- The first question will ask you to explain what $\sum_{n=1}^{\infty} a_n = L$ means (i.e., give a definition of the limit of an infinite series, by invoking partial sums).
- The last question will be an extra credit problem, with some true/false questions.
- I was very impressed with your performance on the first midterm, and will be expecting greatness. Do not be lulled into a false sense of security.

Topics covered on the exam

- Define $\sum_{n=1}^{\infty} a_n = L$.
- Determine whether a series converges by using
  - the n^{th} term test,
  - the integral test,
  - comparison test,
  - limit comparison test,
  - p-series test,
  - geometric series test,
  - harmonic series test,
  - the root test,
  - the ratio test,
  - alternating series test.

  A wise student may infer that I am not likely to ask about the integral test, or the root test.
- Provide statements of the preceeding tests.
- Determine whether a series converges absolutely; determine whether a series conditionally.
- Given a function, write down the first few terms of its Taylor series (around 0 or around another point a)
- Given a function, write down its Taylor series.
- Approximate an alternating series and estimate the error in your approximation.
- Approximate a Taylor series and estimate the error in your approximation.
- State Taylor's theorem and Lagrange's theorem.
- Determine the interval on which a power series converges.
- Differentiate and integrate a power series term-by-term.

What should I write for an answer?

You should not merely give the answer: you should give an explanation. For full credit, you must **justify you argument**. In particular, if you are claiming that a series converges, you must state which test you are applying.

Memorize the following series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \quad \text{(when } -1 < x < 1\text{)}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\log x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n} = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots \quad \text{(when } 0 < x \leq 2\text{)}$$