

FIRST MIDTERM INFORMATION
TEST GIVEN 1/25/10

1. A tautology is a statement form which is true under any truth assignment to its variables. A contradiction is a statement form which is false under any truth assignment to its variables. Two statement forms are logically equivalent if and only if they have the same truth values under any truth assignment to the variables that appear.

The most common scores for this problem were 8 and 10. The most common error is to not mention variables or truth assignments.

2. Straightforward construction of truth table. Some people made an error in the truth table and answered "neither". I gave 0 for that. If you answered correctly, but I spotted a defect in the truth table, then there was a few points taken off.

3. Same comments as 2. A few points taken off for not clearly identifying the critical rows.

4. $(\forall x \in \mathbb{Z})(\sim x < x)$.
 $(\forall x \in \mathbb{Z})(x > 1 \rightarrow x > 0)$.
 $(\exists x \in \mathbb{Q})(\forall y, z \in \mathbb{Z})(\sim x = y+z)$.

5. I will give perfect HAVE/WANT proofs. This is much more than you need to get full credit on tests, and somewhat more than you need to get full credit on homework.

$(\forall x, y \in \mathbb{Z})(y \neq x^2)$. False. Want Negation, which is $(\exists x, y \in \mathbb{Z})(y = x^2)$. Set (Have) $x = 0, y = 0$. Want $0 = 0^2$. Have $0 = 0^2$ by algebra.

$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x = 2y)$. False. Want Negation, which is $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x \neq 2y)$. Set (Have) $x = 1$. Want $(\forall y \in \mathbb{Z})(1 \neq 2y)$. Let (Have) $y \in \mathbb{Z}$. Want $1 \neq 2y$. Assume (Have) $1 = 2y$. Want \perp . Have $y = 1/2$ by algebra. Have $y \notin \mathbb{Z}$ by algebra. Have \perp .

$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Q})(x < y^2)$. True. Want $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Q})(x < y^2)$. Set (Have) $x = -1$. Want $(\forall y \in \mathbb{Q})(-1 < y^2)$. Let (Have) $y \in \mathbb{Q}$. Want $-1 < y^2$. Have $0 \leq y^2$ by algebra. Have $-1 < y^2$ by algebra.

$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{N})(x > y)$. False. Want Negation, which is $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(x \leq y)$. Let (Have) $x \in \mathbb{Z}$. Want $(\exists y \in \mathbb{N})(x \leq y)$. Set (Have) $y = |x|$. Want $x \leq |x|$. Have $x \leq |x|$ by algebra.

$(\exists x, y \in \mathbb{Z})(y \neq x^2)$. True. Want $(\exists x, y \in \mathbb{Z})(y \neq x^2)$. Set (Have) $x = 1, y = 2$. Want $2 \neq 1^2$. Have $2 \neq 1^2$ by algebra.

3 points for the right answer (true or false). 5 points for the right answer and a good justification. 4 points for the right answer and a partial but flawed justification. 0 points for the wrong answer.

6. Most common mistake was to make a circle (even a little one) for Logic. It must be a dot. A few points off for that error.

7.

1. P
2. $p \rightarrow (q \wedge r)$
3. $\neg r \vee s$
4. $(q \wedge s) \rightarrow u$
- $\therefore u$
5. $q \wedge r$ 1,2
6. q 5
7. r 5
8. s 3,7
9. $q \wedge s$ 6,8
10. u 4,9

Most common error is to not break up $q \wedge r$. A few points off for that.

WARNING: Second midterm and final are much more demanding. Some students do very well on the first midterm, but not well on the second midterm. All scores are curved at the end of the course, with a target median final grade of C+.

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54
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41
22

31 students (some students may have dropped)
73 median