3.15. Some Observations.

Recall the Template and Extended Template introduced at the beginning of this Chapter before section 3.1. We now justify the claims TEMP 1,2, and ETEMP, which were also presented there.

TEMP 1. Every one of the 6561 assertions in the Template is either provable or refutable in SMAH+. There exist 12 assertions in the Template, provably equivalent in RCA_0, such that the remaining 6549 assertions are each provable or refutable in RCA_0. Furthermore, these 12 are provably equivalent to the 1-consistency of SMAH over ACA′ (Theorem 5.9.11).

To see how the Annotated Table of section 3.14 justifies Temp 1, recall how it was constructed. The ordered pairs of clauses in the Annotated Table comprise a list of representatives from each equivalence class of the ordered pairs of clauses under the equivalence relation used in section 3.1.

The entries that correspond to the assertions in the Template are the entries in the Annotated Table with INF or ¬INF. The 12 Exotic Cases (see Definition 3.1.2) correspond to the single entry in 28 under ACBC, INF. Every entry in the Annotated Table, with the sole exception of this single entry for INF, was justified in sections 3.3 - 3.13. All of the arguments in sections 3.3 - 3.13 were conducted within RCA_0.

This single entry for INF, corresponding to the 12 Exotic Cases, is equivalent to

PROPOSITION A. For all f,g ∈ ELG there exist A,B,C ∈ INF such that

$$A \cup. fA \subseteq C \cup. gB$$
$$A \cup. fB \subseteq C \cup. gC.$$

Proposition A is the Principal Exotic Case - a particular one of the 12 Exotic Cases that we have chosen on aesthetic grounds. According to Theorem 5.9.11, Proposition A, and hence all 12 Exotic Cases, are provably equivalent to 1-Con(SMAH) over ACA′.

TEMP 2. Every one of the 6561 assertions in the Template, other than the 12 Exotic Cases, are provably equivalent, in

RCA_0, to the result of replacing ELG by any of ELG ∩ SD, SD, EVSD. All 12 Exotic Cases are refutable in RCA_0 if ELG is replaced by SD or EVSD (Theorem 6.3.5).

The first claim of TEMP 2 is justified by the way we derived each entry in the Annotated Table other than 28 under ACBC, INF. Namely, when deriving INF, we always assumed f,g ∈ EVSD rather than f,g ∈ ELG. Note that ELG, ELG ∩ SD, SD ⊆ EVSD. Also see Theorem 3.1.1.

ETEMP. Every assertion in the Extended Template, other than the 12 Exotic Cases with INF, is provable or refutable in RCA_0.

Clearly ETEMP follows from the observation that all of the derivations in this Chapter are conducted in RCA_0. Consideration of the Exotic Cases with INF is postponed to Chapters 4-6.

BRT TRANSFER. Let X,Y,V,W,P,R,S,T be among the letters A,B,C. The following are equivalent.
i. for all f,g ∈ ELG and n ≥ 1, there exist finite A,B,C ⊆ N, each with at least n elements, such that X ∪. fY ⊆ V ∪. gW, P ∪. fR ⊆ S ∪. gT.
ii. for all f,g ∈ ELG, there exist infinite A,B,C ⊆ N, such that X ∪. fY ⊆ V ∪. gW, P ∪. fR ⊆ S ∪. gT.

THEOREM 3.15.1. BRT transfer is provably equivalent to 1-Con(SMAH) over ACA'. Furthermore, BRT forward transfer (i → ii) is provably equivalent to 1-Con(SMAH) over ACA'. BRT backward transfer (ii → i) is provable in RCA_0. Furthermore, BRT forward transfer for the Exotic Cases is provably equivalent to 1-Con(SMAH) over ACA', and BRT forward transfer for ordered pairs other than the Exotic Cases, is provable in RCA_0.

Proof: As entered in the Annotated Table, A ∪. fA ⊆ C ∪. gB, A ∪. fB ⊆ C ∪. gC has ALF, provably in RCA_0. Hence BRT forward transfer, for the Exotic Cases, is provably equivalent, in RCA_0, to A ∪. fC ⊆ C ∪. gB, A ∪. fB ⊆ C ∪. gC has INF. I.e., BRT forward transfer, for the Exotic Cases, is provably equivalent, in RCA_0, to Proposition A. Hence BRT Forward transfer, for the Exotic Cases, is provably equivalent, in ACA', to 1-Con(SMAH).

BRT forward transfer, for other than the Exotic Cases, and BRT backward transfer, are seen, by inspection of the

Annotated Table, to be true. Since the Annotated Table was constructed within RCA0, the remainder of Theorem 3.15.1 has been established. QED

There are some other notable facts concerning the Annotated Table. Recall the obvious implications between our five attributes:

ALF → AL → NON.
ALF → FIN → NON.
INF → AL → NON.

We have also discussed the observed Transfer Property:

INF → ALF → INF.

Are there any other observations to be made from the annotated tables?

Here is the compilation of all attribute lists that are compatible with the above implications:

INF. AL. ALF. FIN. NON.
¬INF. AL. ¬ALF. FIN. NON.
¬INF. AL. ¬ALF. ¬FIN. NON.
¬INF. ¬AL. ¬ALF. FIN. NON.
¬INF. ¬AL. ¬ALF. ¬FIN. NON.
¬INF. ¬AL. ¬ALF. ¬FIN. ¬NON.

All of these are realized from the annotated table:

SINGLE CLAUSES

1. A ∪. fA ⊆ A ∪. gA. ¬INF. ¬AL. ¬ALF. ¬FIN. ¬NON.
3. B ∪. fA ⊆ A ∪. gA. ¬INF. AL. ¬ALF. ¬FIN. NON.
6. A ∪. fA ⊆ B ∪. gA. INF. AL. ALF. FIN. NON.

ABAB

1. A ∪. fA ⊆ B ∪. gA, A ∪. fA ⊆ B ∪. gB. ¬INF. ¬AL. ¬ALF. FIN. NON.
34. C ∪. fA ⊆ B ∪. gA, C ∪. fA ⊆ B ∪. gB. ¬INF. AL. ¬ALF. FIN. NON.

AABA

32. B ∪. fA ⊆ A ∪. gA, B ∪. fB ⊆ A ∪. gB. ¬INF. ¬AL. ¬ALF. ¬FIN. NON.

So there are no more implications between the attributes, in the context of this Chapter.