Algebraic Topology (topics course) **Spring 2010**

Course Information

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Lecture:	Friday 8:15-10, MAA 331
Exercises:	Friday 1:15–3, MAA 331
Office hours:	Monday 4-5, BCH 5115

Welcome. This course is intended for masters students and serious undergraduates. We will develop in detail some of Quillen's spectacular results on the interactions between homotopy theory and algebra. In particular, we will study Quillen's derived abelianization notion of homology, with an emphasis on the André-Quillen homology of commutative algebras.

Underlying Quillen's arguments is a homotopy-theoretic framework to build and control resolutions in non-abelian contexts. The theory is completely flexible and has applications in both geometric and algebraic contexts, such as to the study of the rational homotopy theory of topological spaces, to be treated in this course if time permits.

Objective. Your objective in this course is to learn the basics of homotopy theory in the context of model categories, and to understand in detail some of the interactions between homotopy theory and algebra. In particular, to study in detail Quillen's notion of homology of commutative algebras in terms of derived abelianization. This includes the development of a good understanding of the following:

- construction of the homotopy (or derived) category of a model category
- construction and calculation of (left and right) derived functors
- classical examples of (left and right) derived functors
- homotopy limits and homotopy colimits (homotopy gluing constructions)
- homotopy theory of several geometric and algebraic examples: spaces, G-spaces, chain complexes, simplicial groups, simplicial modules, and simplicial commutative algebras
- resolutions in homotopy theory
- derived abelianization in several geometric and algebraic contexts
- basic properties of André-Quillen homology
- Quillen's results on the rational homotopy theory of spaces (if time permits)

References. The required references for this course are the papers [1, 3, 5] and the suggested references are [2, 4]. The book [6] provides a concise introduction to several algebraic and geometric topics relevant to this course.

References

[1] W. G. Dwyer and J. Spaliński. Homotopy theories and model categories. In *Handbook of algebraic topology*, pages 73–126. North-Holland, Amsterdam, 1995.

[2] P. G. Goerss and J. F. Jardine. *Simplicial homotopy theory*, volume 174 of *Progress in Mathematics*. Birkhäuser Verlag, Basel, 1999.

[3] P. G. Goerss and K. Schemmerhorn. Model categories and simplicial methods. In *Interactions between homotopy theory and algebra*, volume 436 of *Contemp. Math.*, pages 3–49. Amer. Math. Soc., Providence, RI, 2007.

[4] P. S. Hirschhorn. *Model categories and their localizations*, volume 99 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2003.

[5] S. Iyengar. André-Quillen homology of commutative algebras. In *Interactions between homotopy theory and algebra*, volume 436 of *Contemp. Math.*, pages 203–234. Amer. Math. Soc., Providence, RI, 2007.

[6] J. P. May. *A concise course in algebraic topology*. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1999. Available at: `http://www.math.uchicago.edu/~may/` .