Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**
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Handout 4

If $\mathbb{F}$ is any field, one defines the concept of a *vector space over* $\mathbb{F}$ analogously to the notion of a vector space over $\mathbb{C}$ — replace $\mathbb{C}$ by $\mathbb{F}$ everywhere. It is probably better to once again write out the whole definition.

Definition 1. A triple $(V, +, \cdot)$ consisting of a set V and two maps

$$V \times V \xrightarrow{+} V, \qquad (x,y) \longmapsto x + y \qquad \text{“addition”}$$

$$\mathbb{F} \times V \xrightarrow{\cdot} V, \qquad (a,x) \longmapsto ax \qquad \text{“scalar multiplication”}$$

is called a *vector space over* $\mathbb{F}$ if the following eight axioms hold for the maps $+$ and $\cdot$:

(1) $(x+y)+z = x+(y+z)$ for all $x, y, z \in V$.
(2) $x+y = y+x$ for all $x, y \in V$.
(3) There exists an element $0 \in V$ with $x + 0 = x$ for all $x \in V$.
(4) For each $x \in V$ there exists an element $-x \in V$ with $x + (-x) = 0$.
(5) $a(bx) = (ab)x$ for all $a, b \in \mathbb{F}$, $x \in V$.
(6) $1x = x$ for all $x \in V$.
(7) $a(x+y) = ax + ay$ for all $a \in \mathbb{F}$, $x, y \in V$.
(8) $(a+b)x = ax + bx$ for all $a, b \in \mathbb{F}$, $x \in V$.