Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**
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Warm-up Questions 5

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. Let $A \in \mathsf{M}(2 \times 3, \mathbb{F})$, $B \in \mathsf{M}(2 \times 3, \mathbb{F})$. Then
(a) $A + B \in \mathsf{M}(2 \times 3, \mathbb{F})$.
(b) $A + B \in \mathsf{M}(4 \times 6, \mathbb{F})$.
(c) $A + B \in \mathsf{M}(4 \times 9, \mathbb{F})$.

Question 2. For which of the following 3×3 matrices A do we have $AB = BA = B$ for all $B \in \mathsf{M}(3 \times 3, \mathbb{F})$?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Question 3. For $A \in \mathsf{M}(m \times n, \mathbb{F})$, we have
(a) A has m rows and n columns.
(b) A has n rows and m columns.
(c) The rows of A have length m and the columns of A have length n.

Question 4. Which of the following matrix products is zero?

(a) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$

Question 5. Which of the following properties does matrix multiplication lack?

(a) associativity (b) commutativity (c) distributivity

Question 6. For $A \in \mathsf{M}(n \times n, \mathbb{F})$ we have:
(a) $\operatorname{rk} A = n \Rightarrow A$ is invertible, but there exist invertible matrices with $\operatorname{rk} A \neq n$.
(b) A is invertible $\Rightarrow \operatorname{rk} A = n$, but there exist matrices A with $\operatorname{rk} A = n$, which are not invertible.
(c) $\operatorname{rk} A = n \Leftrightarrow A$ is invertible.

Question 7. Let $A \in \mathsf{M}(m \times n, \mathbb{F})$, $B \in \mathsf{M}(n \times m, \mathbb{F})$, so that we have

$$\mathbb{F}^n \xrightarrow{A} \mathbb{F}^m \xrightarrow{B} \mathbb{F}^n.$$

Let $BA = I$ (which equals $\mathrm{Id}_{\mathbb{F}^n}$ as a linear map). Then
(a) $m \geq n$, A injective, B surjective.
(b) $m \leq n$, A surjective, B injective.
(c) $m = n$, A and B invertible (bijective).

Question 8. For $A \in \mathsf{M}(m \times n, \mathbb{F})$ with $m \leq n$, we always have

(a) $\operatorname{rk} A \leq m$ (b) $m \leq \operatorname{rk} A \leq n$ (c) $n \leq \operatorname{rk} A$