Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**

John E. Harper
Purdue University

Handout 2

Recall from lecture that instead of calculating in the field of real numbers $\mathbb{R}$ we may want to calculate in the field of complex numbers $\mathbb{C}$. This leads to the following notion of a "complex vector space", also called a "vector space over $\mathbb{C}$", which is defined analogously to a real vector space: one has only to replace $\mathbb{R}$ by $\mathbb{C}$ and "real" by "complex" in all instances.

Definition 1. A triple $(V, +, \cdot)$ consisting of a set V and two maps

$$V \times V \xrightarrow{+} V, \qquad (x, y) \longmapsto x + y \qquad \text{"addition"}$$

$$\mathbb{C} \times V \xrightarrow{\cdot} V, \qquad (a, x) \longmapsto ax \qquad \text{"scalar multiplication"}$$

is called a *complex vector space* (or *vector space over* $\mathbb{C}$) if the following eight axioms hold for the maps $+$ and $\cdot$:

(1) $(x + y) + z = x + (y + z)$ for all $x, y, z \in V$.
(2) $x + y = y + x$ for all $x, y \in V$.
(3) There exists an element $0 \in V$ with $x + 0 = x$ for all $x \in V$.
(4) For each $x \in V$ there exists an element $-x \in V$ with $x + (-x) = 0$.
(5) $a(bx) = (ab)x$ for all $a, b \in \mathbb{C}$, $x \in V$.
(6) $1x = x$ for all $x \in V$.
(7) $a(x + y) = ax + ay$ for all $a \in \mathbb{C}$, $x, y \in V$.
(8) $(a + b)x = ax + bx$ for all $a, b \in \mathbb{C}$, $x \in V$.