Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**
John E. Harper
Purdue University

Warm-up Questions 3

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. For which of the following objects does the description “linearly dependent” or “linearly independent” make sense?

(a) An n-tuple $(v_1, \ldots, v_n)$ of elements of a vector space
(b) An n-tuple $(v_1, \ldots, v_n)$ of real vector spaces
(c) A linear combination $c_1v_1 + \cdots + c_nv_n$

Question 2. Let V be a vector space over a field $\mathbb{F}$. Let $v_1, \ldots, v_n \in V$. What does the assertion that $\mathrm{Span}(v_1, \ldots, v_n) = V$ mean precisely?

(a) Each linear combination $c_1v_1 + \cdots + c_nv_n$ is an element of V.
(b) Each element of V is a linear combination $c_1v_1 + \cdots + c_nv_n$.
(c) The dimension of V is n.

Question 3. Let V be a vector space over a field $\mathbb{F}$. For linearly independent triples (v_1, v_2, v_3) of vectors in V,

(a) (v_1, v_2) is always linearly dependent.
(b) (v_1, v_2) may or may not be linearly dependent, depending on the choice of (v_1, v_2, v_3).
(c) (v_1, v_2) is always linearly independent.

Question 4. Let $\mathbb{F}$ be a field. The i-th vector of the canonical basis of $\mathbb{F}^n$ is defined by

(a) $e_i = (0, \cdots, i, \cdots, 0)$.
(b) $e_i = (0, \cdots, 1, \cdots, 0)$.
(c) $e_i = (1, \cdots, 1, \cdots, 0)$.

Question 5. Let V be a vector space over a field $\mathbb{F}$. Which of the following statements implies the linear independence of the n-tuple $(v_1, \ldots, v_n)$ of elements of V?

(a) $c_1v_1 + \cdots c_nv_n = 0$ only if $c_1 = c_2 = \cdots = c_n = 0$.
(b) If $c_1 = \cdots = c_n = 0$, then $c_1v_1 + \cdots + c_nv_n = 0$.
(c) $c_1v_1 + \cdots + c_nv_n = 0$ for all $(v_1, \ldots, v_n) \in \mathbb{F}^n$.

Recall from lecture the following proposition.

Proposition 1. *Let V be a finite dimensional vector space over a field $\mathbb{F}$. Any linearly independent finite ordered set $L \subset V$ can be extended by adding elements, to get a basis.*

Question 6. What does Proposition 1 and its proof imply in the case that $L = \emptyset$?

(a) If $(w_1, \cdots, w_s)$ is an s-tuple of vectors in V and $\mathrm{Span}(w_1, \cdots, w_s) = V$, then one can extend $w_1, \cdots, w_s$ to a basis.

(b) If $(w_1, \cdots, w_s)$ is a linearly independent s-tuple of vectors in V, then there exists a basis consisting of vectors from $(w_1, \cdots, w_s)$.
(c) If $(w_1, \cdots, w_s)$ is an s-tuple of vectors in V and $\mathrm{Span}(w_1, \cdots, w_s) = V$, then there exists a basis consisting of vectors from $(w_1, \cdots, w_s)$.

Question 7. The vector space $V = \{0\}$ over a field $\mathbb{F}$ consisting of the zero element
(a) has the basis (0).
(b) has the basis $\emptyset$.
(c) has no basis.

Question 8. Let V be a vector space over a field $\mathbb{F}$. If one were to define

$$U_1 - U_2 := \{x - y \mid x \in U_1, y \in U_2\}$$

for subspaces U_1, U_2 of V, then one would have
(a) $U_1 - U_1 = \{0\}$.
(b) $(U_1 - U_2) + U_2 = U_1$.
(c) $U_1 - U_2 = U_1 + U_2$.

Question 9. Let V be a vector space over a field $\mathbb{F}$. For subspaces U_1, U_2 of V, one always has
(a) $(U_1 + U_2) + U_3 = U_1 + (U_2 + U_3)$.
(b) $U_1 \cap (U_2 + U_3) = (U_1 \cap U_2) + (U_1 \cap U_3)$.
(c) $U_1 + (U_2 \cap U_3) = (U_1 + U_2) \cap (U_1 + U_3)$.

Question 10. Let V be a vector space over a field $\mathbb{F}$. Subspaces U_1, U_2 of V are said to be *transverse* (to each other) if $U_1 + U_2 = V$. One calls $\operatorname{codim} U := \dim V - \dim U$ the *codimension* of U in V. For transverse U_1, U_2, one has
(a) $\dim U_1 + \dim U_2 = \dim U_1 \cap U_2$.
(b) $\dim U_1 + \dim U_2 = \operatorname{codim} U_1 \cap U_2$.
(c) $\operatorname{codim} U_1 + \operatorname{codim} U_2 = \operatorname{codim} U_1 \cap U_2$.