Math 254
Autumn 2003

Review 1

(1.) Determine if the following limits exist. Find the limit if it does exist.

(a.) $\lim_{(x,y,z)\to(1,-1,2)} \dfrac{x^2yz - 2xyz}{x^2 + y^2 - z^2}$ **(b.)** $\lim_{(x,y)\to(0,0)} \dfrac{x^2 \sin x}{x^2 + y^2}$ **(c.)** $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2 + y^2}$

(2.) Find all first partial derivatives for the given function (Do not simplify):

(a.) $f(x,y) = [\ln(x^2 + 2xy)]^2$ **(b.)** $f(x,y,z) = \sqrt{xyz}(x^2 - 2z)$ **(c.)** $f(x,y) = \frac{\sin(x^2y)}{x^2 - y}$

(3.) Find the indicated (partial) derivatives of the given function (Do not simplify):

(a.) $w = u^2 + 2uv^2, u = t^3, v = 2t + \ln t$; $\frac{dw}{dt}$
(b.) $w = \sin(uv) - \cos(2uv), u = x^2 + xy, v = y^2 + 2xy$; $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$
(c.) $f(x,y) = x^2 - 2xy + y^2 + 4x - 8y - 1$; $f_{xx}, f_{xy}, f_{yx}, f_{yy}$

(4.) Suppose that $w = f(x - y, y - z, z - x)$. Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

(5.) Find the directional derivative of $f(x,y,z) = 4 - x^2 + 3y^2 + y + z$ in the direction of $\mathbf{u} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ at the point $(-1,0,0)$.

(6.) Find the gradient of $f(x,y,z) = xye^z$ at the point $(2,3,0)$.

(7.) Find a vector which is normal to the surface $x^2 - 3x^2y + 2y^3 = 11$ at the point $(-1,2)$.

(8.) Find a vector which is normal to the graph of $f(x,y) = ye^x$ at the point $(0,1,1)$

(9.) Find the equation of the tangent plane to the surface $x^2 - y^2 - z^2 = 1$ at the point $(3,-2,-2)$.

(10.) Find all critical points, relative maximums/minimums and saddle points of the given functions:

(a.) $f(x,y) = x^2 + y - \frac{1}{2y^2}$ **(b.)** $-2x^2 + xy + y^2 - 4x + 3y - 1$

(11.) Find the extreme values of $f(x,y) = x^2 + y^2 - 2x - 4y - 6$ over the disk $x^2 + y^2 \le 16$.

(12.) Suppose that two non-overlapping rectangles are placed in the triangle whose vertices are $(0,1), (1,0)$ and $(0,0)$. If their sides are parallel to the coordinate axes, how large can their total area be? See figure below (hint: find a relationship between a and b and another between c and d.)