Math 571
Autumn 2003

Review 1

(1.) Consider the following system of linear equations:

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 + 2x_4 + x_5 &= 12 \\ 3x_1 - 4x_2 - 2x_3 + x_4 &= 1 \\ 4x_1 + 5x_2 + x_3 - 6x_4 &= 21 \\ x_1 - x_2 - 2x_3 - x_4 + 3x_5 &= 1 \\ 2x_1 - 2x_2 + x_4 - x_5 &= 0 \end{aligned}$$

(a.) Find the augmented matrix of this system.
(b.) Find the reduced row echelon form of the matrix in (a.).
(c.) Is this system consistent? If so, find all solutions.

(2.) Let $A = \begin{pmatrix} 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 3 \\ -1 & -2 & 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ -1 & 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$.

Find AB, BA, BC, $A + C$ and $B + C$.

(3.) Given the augmented matrix A= $\left(\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 3 & 2 & -2 & -2 \\ -2 & 6 & \alpha & \beta \end{array}\right)$, find all values of α and β such that the corresponding linear system has:

(a.) no solutions,
(b.) a unique solution or
(c.) infinitely many solutions.

(4.) Find an elementary matrix $\mathcal{E}$ such that $\mathcal{E}A = B$ where:

(a.) $A = \begin{pmatrix} 2 & -2 & 3 \\ -1 & 4 & 2 \\ 3 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -2 & 3 \\ 3 & -12 & -6 \\ 3 & 1 & 2 \end{pmatrix}$ **(b.)** $A = \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 5 & 3 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 7 & 7 & 7 \end{pmatrix}$

(5.) True or False. You do not need to explain your answer.

(a.) If A and B are $n \times n$ matrices, then $\det(A + B)$=$\det(A)$+$\det(B)$
(b.) If A and B are $n \times n$ matrices, then $AB = BA$
(c.) Suppose $A\mathbf{x} = \mathbf{b_1}$ has a unique solution, then it is possible for $A\mathbf{x} = \mathbf{b_2}$ has more than one solution.
(d.) An underdetermined system of linear equations is always consistent.
(e.) Every $n \times n$ elementary matrix is invertible.

(6.) Find the inverse and determinant of the given matrices:

(a.) $\begin{pmatrix} 2 & 5 & -2 \\ 1 & -1 & 4 \\ 3 & 0 & 1 \end{pmatrix}$ **(b.)** $\begin{pmatrix} 1 & 3 & 1 & 2 \\ -1 & 2 & 0 & 1 \\ 0 & 5 & -1 & -1 \\ 0 & -1 & 2 & 0 \end{pmatrix}$

(7.) Prove the following:

(a.) If A is invertible, then $\det(A^{-1})$=$1/\det(A)$
(b.) If A is invertible, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$
(c.) If A is invertible, then $A = \mathcal{E}_1\mathcal{E}_2 \cdots \mathcal{E}_k$ where $\mathcal{E}_1, \mathcal{E}_2, \ldots \mathcal{E}_k$ are elementary matrices.