Math 571

Autumn 2009

Quiz 6

Instructions: This quiz is worth 10 points and the value of of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(3 pts.) Let S be the subspace of $\mathbb{R}^5$ spanned by $\mathbf{x}_1 = (1, 5, -1, 3, 2)^T, \mathbf{x}_2 = (3, 3, 1, 1, -10)^T$ and $\mathbf{x}_3 = (7, 5, 13, -9, 4)^T$.

- **(a.)** Verify that $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is an orthogonal basis for S.
- **(b.)** Find the orthogonal projection $\mathbf{p}$ of $\mathbf{y} = (1, -2, 2, -3, 4)^T$ onto S.

(2.)(3 pts.)Let V be the vector space $C[-1, 1]$ with the usual inner product and let S be the subspace of V spanned by $\mathbf{x}_1 = 1, \mathbf{x}_2 = x$ and $\mathbf{x}_3 = 3x^2 - 1$

- **(a.)** Verify that $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is an orthogonal basis for S.
- **(b.)** Find the orthogonal projection $\mathbf{p}$ of $\mathbf{y} = \sqrt[3]{x}$ onto S.

(3.)(2 pt.) Let S be the subspace of $\mathbb{R}^4$ with $S = \text{Span}\{(1, 1, -2, 1)^T, (3, 1, 6, -2)^T\}$. Find a basis for $S^\perp$.

(4.)(2 pts.) Find the least squares solution $\hat{\mathbf{x}}$ to the system: $\begin{pmatrix} 1 & 4 & 5 \\ 3 & 1 & 2 \\ 2 & 1 & 10 \\ -1 & -1 & -3 \\ -3 & 2 & 5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 3 \\ -2 \\ 7 \\ 1 \end{pmatrix}$